III. Electrical Generation

Field Currents

In the nervous system, extracellular electrical fields arise only in conjunction with moving ions. Current is the change of charge with time.

\[ i = \frac{dq}{dt} \]

A **source** is somewhere that moving charge comes from.
A **sink** is somewhere that moving charge goes to.
Because neurons can neither create nor destroy charge, there are no true sources and sinks in the brain. There are only two other possibilities.

(1) The charge is pumped from one region to another with changes in charge density in those regions. This is not possible because neurons cannot generate large enough forces to alter charge densities in aqueous solution, except at the membrane where there is local separation of ion species. Away from the membrane, positive and negative charge balance out in any arbitrarily small volume. Even at the membrane, the local separation of charge does not change overall charge density.
(2) The charge is moved in a closed loop. The neuron consists of a closed surface (the membrane) across which ionic currents flow in closed loops.

- The total current across any closed surface such as the membrane is always zero.
- Current in must equal current out.
- The membrane becomes a boundary for the inner and outer fields. An area of current outflow is a source for the outer field and a sink for the inner field. The reverse holds for an area of current inflow.
- Because the total transmembrane current is zero, the sum of apparent sources equals the negative of the sum of apparent sinks (for both the inner and outer fields).
Topological model

• The internal and external compartments and currents have three dimensions, and there is great variability in their geometries.
• In 2 dimensions the structure and flow can be represented by a closed boundary and by closed lines of current in a topological model.
• Transmembrane current $i_m$ flows across the membrane and longitudinal current $i_i$ and $i_e$ flows parallel to the membrane.
Axon Fields
(1) External Potential
Lorente de Nó (1947): Place a cut nerve on a piece of blotter paper soaked in saline solution. Record action potentials at different points in external field. Gives 2-dimensional map.
The problem can be simplified by suspending the nerve fiber in air or oil. This leaves only a thin sheath of conducting fluid around the nerve, thus constraining the external field to a single dimension. The external potential function is a mirror image of the internal potential function in this case. The distribution of potential with distance $v(x)$ is a monophasic, negative function.
(2) Longitudinal Current
The first derivative (gradient) of potential with respect to distance is biphasic. It is maximal and minimal at the sites of maximal rising and falling slopes in potential, and it is zero at the site of maximal potential.

How do we explain this distribution? From Ohm's law we know that the gradient of potential equals the product of the longitudinal resistance with the longitudinal current $i_l$.

$$\frac{dv(x)}{dx} = r \cdot i_l$$

Thus the biphasic plot of the gradient is proportional to the longitudinal current. This shows that the longitudinal current is zero at the site of maximal current influx, and travels in opposite directions away from this point.
(3) Transmembrane Current
The second derivative (divergence) of potential with respect to distance is triphasic. Its minimum is at the point where the gradient is zero. It crosses zero at the gradient's maximum and minimum. Its maxima are at the points of maximal change of the gradient. How do we explain this distribution?
From Kirchhoff's current law, the sum of currents entering and leaving every node at all times is zero:

$$\sum_{n} i_n(t) = 0$$
For any arbitrarily small segment of axon, the difference between the longitudinal current entering and leaving the segment must equal the transmembrane current in that segment. There is no place else for it to go or come from. This tells us that for each point on the axon, the rate of change of longitudinal current is equal to the transmembrane current density:

\[ j_m = \frac{d i_l}{dx} \]
Combining this result with Ohm's law gives:

\[ j_m = \frac{1}{r} \frac{d^2 v(x)}{d x^2} \]

Thus the second spatial derivative of potential is proportional to the transmembrane current density as the action potential travels past the recording electrode.

The minimum of the second spatial derivative reflects the current sink of the action potential, and the two maxima reflect its leading and trailing sources.