2.1. INTRODUCTION

The data arising from an electrophysiological experiment on the nervous system initially consist of records in continuous analog form of stimulus events and the responses that they give rise to. If these data are to be analyzed in more than a qualitative way, digital computation techniques are usually called for. This means that the analog data have first to be converted to digital, sampled form. Then the full range of analysis techniques that have been developed to study dynamic processes can be brought to bear. These include filtering, averaging, spectral analysis, and covariance analysis. In this chapter we discuss first the properties of the analog-to-digital conversion processes with particular regard to their effect on the experimental data, and the subsequent tests the data are subjected to. Then we move to a discussion of filtering operations, analog and digital, with emphasis on the latter and how it fits into computer data analysis procedures. From time to time we consider some of the hardware aspects of filtering since familiarity with them is quite useful for a fuller comprehension of filtering procedures.

2.2. ANALOG-TO-DIGITAL CONVERSION

An analog-to-digital converter (ADC) converts a continuous signal into a sequence of T- and A-discrete measurements. The two steps of time sampling and amplitude quantizing are usually performed in a combined procedure. The ADC is first given the command to sample by the computer and then holds the amplitude of this sample briefly while quantizing it. We illustrate the ADC in Fig. 2.1 as performing its operations in the sequence...
sequence of maintained voltage levels lasting the duration between sampling times, Fig. 2.1(b). The amplitude of each level is the signal amplitude at the sampling instant \( t^o \). In what follows, we assume \( \Delta \) to be unity so that \( t^o \Delta \) can be replaced by the integer valued time variable \( t^o \). Sampling devices are often referred to as sample-and-hold circuits because of their ability to hold the sampled value without significant decay until quantization has been completed—a time duration that is often considerably shorter than the interval between samples.

In a number of experimental situations in which a response to a stimulus is being analyzed, the instrumentation is organized so that the stimulator is triggered by the same pulse that initiates A-D conversion of the data. This insures that there will be no jitter (random variation in time) or asynchrony between the onset of the stimulus and the data sampling instants. That is, sampling always occurs at fixed delays from stimulus onset. If, on the other hand, the stimulator is driven independently of the ADC and notifies that device when to initiate sampling, jitter of the sampling instants can occur and tend to result in some temporal smearing of the digitized data. The jitter effect will be small when the cycle time of the computer is small compared with the sampling interval. Here we ignore the effects of jitter in A-D conversion.

The sampled signal \( x_a(t) \) is then quantized to yield an output \( x_q(t^o) \) which can take on only a limited number of, usually, uniformly spaced values. The input-output relationship for the quantizer is shown in Fig. 1(c). The quantization step is \( q \) volts in amplitude. The output is 0 as long as the input is greater than 0 and no larger than \( q \); it is \( q \) as long as the input is greater than \( q \) and no larger than \( 2q \) and so on. In equation form, the input-output relationship is, at integral values of \( t = t^o \) (with \( \Delta = 1 \))

\[
x_q(t^o) = \begin{cases} 
Mq, & \text{for } x_a(t^o) \geq Mq = Q \\
mq, & \text{for } m q < x_a(t^o) \leq (m + 1)q, \quad |m| \leq M \\
-Mq, & \text{for } x_a(t^o) < -Mq = -Q 
\end{cases}
\]
The maximum and minimum voltage levels that can be handled without saturation are $Q$ and $-Q$ and the total number of levels $2M$ that the output signal can take on is usually some integer power $L$ of 2:

$$2M = 2^L$$  \hfill (2.2)

The degree of precision of an A-D conversion is referred to in terms of the number of bits in the output word of the converter. A 10-bit converter will quantize voltages between $-1$ and $+1$ Volt into one of 1024 levels each of whose magnitude is 1.952 mV.

The final step in the conversion is to code $x_q(t^n)$ (only the values of $x_q$ at the sampling times are important) into a form acceptable for use by the digital computer. Most often this means that $x_q(t^n)$, whether positive or negative, is represented in binary form, $L$ binary digits being adequate for this. Typically, one coded output line is assigned to each binary digit and the value of the voltage on this line at the read-out time indicates whether that binary digit is a 1 or a 0. The time for both sampling and read-out are determined by a clock contained within the computer. "Interrupt" features of the computer assure that the incoming data are accepted after each quantization has been performed.

2.3. QUANTIZATION NOISE

Each conversion has associated with it a discrepancy between the quantized and the true value of the signal. It is useful to consider this error as a form of noise, called quantizing noise, $z_q(t^n)$. We can then write

$$x_q(t^n) = x(t^n) + z_q(t^n)$$  \hfill (2.3)

$z_q$ is limited in absolute value to 1/2 the size of the quantizing step $q$. (The properties of quantizing noise in the uppermost and lowermost quantizing levels are different but do not substantially alter this analysis.) We assume the incoming signal to be a random one that is band limited to $F = 1/2$ such that $\Delta = 1$. This means that sampling is done at the Nyquist rate. We also assume that the signal's amplitude is large compared to the size of a quantizing step but small enough not to produce peak value limiting at any time in the converter. Under these reasonable assumptions the following statements hold reasonably well: (1) the quantizing error of a sample is uncorrelated with that of its sequential neighbors; (2) the probability density function for the error $z_q$ of a sample is uniformly distributed over the interval 0 to $q$. That is, it is equally likely that the magnitude of the error be anywhere in this range. From assumption (2) and the quantization rule of Eq. (2.1), the mean value of the quantizing noise is $q/2$. This is a bias term.

$$\text{var}[z_q] = \int q/2 z_q^2 dz = \frac{q^2}{12}$$  \hfill (2.4)

The lack of correlation between sample errors implies that the autocovariance function for the noise is given by

$$c_{z_q z_q}(\tau^n) = \begin{cases} \frac{q^2}{12}, & \tau^n = 0 \\ 0, & \text{otherwise} \end{cases}$$  \hfill (2.5)

The power spectrum of the noise, excluding the dc bias term, is flat to $F = 1/2$. To see this, suppose the data consist of $N$ samples of the signal and that we assume the combination of signal and noise to be periodic with period $T = NA = N$. The substitution of Eq. (2.5) into Eq. (1.23) results in spectral terms $c_{z_q z_q}(n)$, which are all equal and independent of $n$. This is because $c_{z_q z_q}(\tau^n)$ is different from 0 only when $\tau^n = 0$. Thus the quantizing noise is equally divided among all the $N/2$ frequency components between 0 and $N/2$:

$$c_{z_q z_q}(n) = \frac{q^2}{12N}, \quad 0 \leq n < N/2$$  \hfill (2.6)

The ADC converter thus adds noise of its own to the incoming signal, a noise whose covariance and spectral properties are determined solely by the sampling rate and the fineness of quantization. Although quantizing noise has the appearance of being random, it is best to remember that this is not entirely so. To illustrate this point, suppose the incoming signal were a repetitive wave synchronized exactly to some multiple of the sampling period. Samples...
taken of the waveform during each period at the same time relative to the beginning of a period will always produce the same quantizing error and this would not be removable by the process of averaging over successive waveform repetitions. However, as soon as some background noise is added to the fixed waveform, the situation changes. The quantizing noise then takes on many of the characteristics of random noise. In a sense, the uncorrelated quantizing noise is induced into the quantized signal whenever the incoming signal has a fluctuating random component. Thus, if the input noise bandwidth were very low relative to 1/2, the quantizing noise would still exhibit the flat power spectrum indicated by Eq. (2.6). This induced noise can only be removed by numerical or digital filtering of the digital data subsequent to the A-D conversion operation, a topic covered later in this chapter. Since there are many situations in which one is interested in signal peaks which may be small compared to the largest one present, the existence of quantizing noise must not be ignored, for it tends to make the small peaks less detectable. It can, for example, become an important factor when the biological noise contains a significant amount of low-frequency components giving rise to what is referred to as baseline drift in the received data. When this occurs, it is common practice to reduce the amplification of the signal so as to prevent too frequent saturation of the signal amplifiers or peak limiting in the ADC. It is then quite possible that lesser peaks in the signal will be no larger than a few quantizing intervals, making the quantizing noise a factor of importance.

The fineness of A-D quantization is of importance in still another way. It affects the ability to reconstruct from the quantized output data, the amplitude probability distribution of the input data. This issue is somewhat different from that of detecting by response averaging a weak but constant response in a background of noise (Chapter 4). There, one is not interested in determining the nature of the amplitude distribution of the data. Here, detection of such subtleties in the data is the desideratum, with response detection being secondary. To find how well this can be done, it is necessary to know how fine, relative to the peaks in the amplitude distribution, the quantization steps must be. When a large number of quantized samples of the input signal are available, the answer, as Tou (1959) has shown, can be arrived at by considering the signal amplitude distribution as itself a waveform which is to be represented by a set of uniformly spaced samples along the amplitude axis. In this approach, the amplitude axis is analogous to the time axis of conventional waveform sampling. One can then apply the sampling theorem that states that for perfect reconstruction of a band limited wave whose highest frequency is \( F \), sampling should be performed at a rate no lower than 2\( F \) sec.

In practice, when the experimenter examines the sampled version of the waveform on an oscilloscope, the Nyquist rate is usually inadequate to permit satisfactory visual reconstruction of the waveform. Sampling rates for this purpose should be no lower than 3\( F \) sec to 5\( F \) sec. Although probability distributions of amplitude are not truly band limited in terms of their Fourier transforms (called characteristic functions), it is possible to arrive at a convenient rule-of-thumb in determining what an adequate quantization step or sampling interval should be. Thus, suppose the narrowest peak in the amplitude probability distribution of the data is normal in shape, with variance \( \sigma^2 \). The Fourier transform of this distribution is also Gaussian and has more than 99% of its area confined to "frequencies" less than 1/3\( \sigma \). Considering this to be an adequate approximation to the "bandwidth" of the distribution, simple computations indicate the size of the quantizing step should then be very nearly \( \sigma \). Note that though quantizing noise is present, its variance, \( \sigma^2/12 \), is small compared to the variance of the smallest peak in the input distribution. Our rule can now be stated in terms of the distance \( D \) between points three standard deviations away from the narrowest peak: a sampling width \( D/6 \) volts is adequate to represent peaks in the amplitude distribution which are \( D \) volts or more in width. The result holds for overlapping
peaks as long as no component peak is narrower than $D$. If the peaks are sharper, the rule stated here will produce some distortion of their shapes which will be further contaminated by quantization noise. Sharp peaks therefore require some decrease in the quantization step.

2.4. MULTIPLEXING: MONITORING DATA SOURCES SIMULTANEOUSLY

Multiplexing is the process whereby several data sources have their information transmitted to the data processor over the same channel. Here the channel is the ADC and the multiplexing is performed by a process of switching the input of the ADC from one signal source to another. The rate at which the switching is performed and the choice of the source to be selected are determined by the data processor which accepts the data from the converter output. Both are constrained, of course, by the data handling capabilities built into the converter. When multiplexing is performed, an additional amount of time is required to perform a data conversion. The additional time arises because the process of switching the data converter from one source to another introduces a brief electrical transient into the signal and it is necessary to wait for this transient to subside before performing a conversion. The multiplexing time can increase the total conversion time by about 10%.

Multiplexing of different data sources is performed most commonly at a uniform rate proceeding from source 1, to source 2, to source 3, etc., and back to source 1 in a recurrent, cyclic fashion. This is the mode of operation when the data from the different sources are signals of comparable bandwidths and whose temporal fluctuations are judged to be of equal interest and importance. When equal sharing of the ADC by the different sources occurs, the minimum period between samples of any one source is increased by a factor equal to the total number of multiplexed channels. As a consequence, the maximum bandwidth which each signal can have without introducing spectral aliasing is $1/2M\omega$ where $M$ is the number of equally multiplexed sources and $1/\omega$ is the effective sampling rate. In addition to being certain that the effective sampling rate is adequate to preserve signal structure, one must also consider the effects of noise in the input data and quantization noise. Ideally, prior to A-D conversion, filtering should be performed to remove from the input data all frequency components higher than $1/2M\omega$. If this is not done, the higher frequency noise components in the data will, after digitizing, be aliased with the lower frequency ones. Aliasing means that signal components at frequencies greater than 1/2 of the sampling rate will be misinterpreted as components at frequencies less than half the sampling rate. This falsifies the interpretation of signal structure made from the sampled data. Aliasing is discussed more thoroughly in Chapter 3. The net result is a decrease in the signal-to-noise ratio of the digitized data. Suppose, for example, that the sampling rate of the ADC were 1000 samples/sec and that five data channels were being multiplexed. Suppose also that the prefilter had a high frequency cutoff at 500 Hz corresponding to the resolvable bandwidth if only one channel were being digitized. Now, five data sources are being multiplexed. The effective sampling rate of each source is 200/sec and the corresponding resolvable bandwidth is 100 Hz. Even if the response components of the input data have bandwidths less than 100 Hz, all the instrument noise between 100 Hz and the filter cutoff at 500 Hz will be aliased into the spectral region below 100 Hz, producing a degradation of the quantized data from the ADC. This degradation can be eliminated only by reducing the input data bandwidth to 100 Hz. For this reason it is highly desirable when background noise is an important factor to use a prefilter whose cutoff frequency is 1/2 the effective sampling rate. The total quantizing noise remains unchanged during multiplexing since the quantizing error in each conversion is the same. However, the bandwidth of the digitized output has been reduced so...
that the spectral intensity of the quantizing noise is increased by the factor $M$. Filtering prior to A-D conversion cannot reduce this. As basic communications theory shows, this means that when sampling is done at the Nyquist rate, narrow bandwidth data are more affected by quantization noise than are broad bandwidth data.

In some situations, the monitored data sources have widely different bandwidths making it possible to sample the narrow bandwidth signals less frequently than the broad. This often results in a nonuniform rate of sampling of the broader bandwidth signals, there being occasional intervals in which they are not sampled. Usually no serious deterioration in the data analysis results. Infrequent interruptions in sampling can be further minimized by post A-D conversion digital filtering, discussed later in this chapter, which has the effect of interpolating the missed data points in addition to smoothing the data.

If one considers only the spectral properties of the data sources and the sampling rate of the ADC, the problems associated with multiplexing are straightforward. However, another factor, the size of the computer storage area, needs also to be considered when real time data analysis is being performed. As discussed previously, in single channel A-D conversion all real-time data processors have a limited memory capacity in terms of the number of registers available to store data. When multiplexing is employed, these registers are parceled out to the different data sources so that over a given observation epoch, it is never possible to attain the same temporal resolution in each of the several multiplexed channels as it is with just one. The decision to resort to multiplexing must take this into account.

2.5. DATA FILTERING

The operation of data filtering is one in which certain attributes of the data are selected for preservation in preference to others which are "filtered out." To design a satisfactory