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Noise improves three-dimensional perception: Stochastic resonance and other impacts of noise to the perception of autostereograms

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Autostereograms can be perceived in different well-defined spatial levels. Therefore they are an excellent tool with which to examine spatiotemporal processes of multistable three-dimensional perception. We study properties of spatial ambiguity such as phase transitions between different spatial levels and hysteresis in perception with and without noise. We show that the perception of physical noise—which is added to the autostereograms in the form of a random dot pattern—is dependent on the perceived spatial level. We demonstrate that noise can be helpful for the perception of depth in some cases. We show that the signal-to-noise ratio of depth perception is enhanced at an intermediate level of noise strength that is the signature of stochastic resonance in depth perception.

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I. INTRODUCTION

We explore the use of autostereograms as an alternative tool to use to understand spatiotemporal processes of depth perception. The recognition process of autostereograms is an excellent technique to use to explore the dynamics of ambiguous depth perception, including memory effects in the brain. Autostereograms can produce a whole cascade of phase transitions between different perceptual states *s*. The physical reason for this different three-dimensional (3D) state is the ambiguity of the stereoscopic correspondence problem in autostereograms connected to their special design.

Autostereograms are a way of encoding 3D information in two-dimensional (2D) pictures in the form of computer generated patterns of colored dots [1–3]. At a first glance, these patterns appear as meaningless structures. After a certain observation time, a 3D pattern suddenly becomes visible in a strikingly perceptual transition. In Fig. 1(a), an example of an autostereogram designed for our experiment is presented. After some time of observing a flat, periodic random dot pattern, a 3D rose suddenly becomes visible.

The human skill of depth perception is a consequence of two different horizontally shifted locations of the two eyes. The 3D world is therefore projected as two different pictures on the retinae. The horizontal difference of the projections of each detail of the 3D structure is called *disparity*. Julesz [4] showed that this disparity information alone is sufficient to perceive depth. In his so-called *random dot stereograms*, he used a pair of random dot pictures with encoded depth information.

In autostereograms, the horizontal length of the random dot pictures is diminished and therefore so is their distance. The smaller the size of the pictures, the more often they can be repeated horizontally. Therefore the stability of 3D perception is enhanced. This was already studied on the so-called *wallpaper pattern illusion*, which was first found in 1844 [5] by watching a wallpaper pattern in a horizontally repeated sequence. All the repetitive patterns of the wallpaper looked the same. Therefore the two eyes can fuse many different combinations of patterns. This leads to different perceived disparities and illusory depths. In the wallpaper illusion, the stereoscopic correspondence problem thus becomes ambiguous in the extreme. It has been shown that there is an clarifying effect with regard to the ambiguous wallpaper illusion. The perceptual system prefers the state with the smaller disparity [6].

Autostereograms consist of random dot stereograms with a certain number of repetitions. Therefore they generalize and combine the wallpaper illusion effect and the random dot stereograms. As soon as there are more than two repetitions, the 3D perception gets ambiguous, and it depends on the fixation points of the two eyes. There is an optimum repetitive design of autostereograms, because a compromise has to be made between two different factors. In order to ease the stereoscopic viewing conditions, the horizontal distance between the repetitive random dot patterns has to be made as small as possible. On the other hand it cannot become too small because of a decreasing 3D resolution.

The encoding scheme of depth is based on the disparity

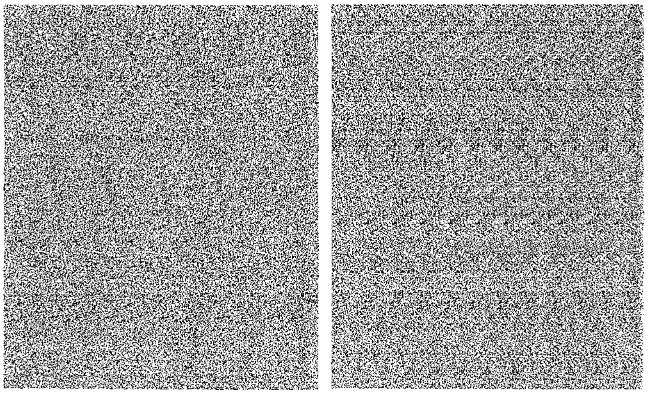

FIG. 1. Example of an autostereogram. (a) (left): Intermediate period lengths ($p=100$ pixel); the stereoscopic state $s=1$ is favored. (b) (right): The same autostereogram with a smaller period length ($p=50$ pixel): the state $s=2$ is favored, after looking to (a) first.

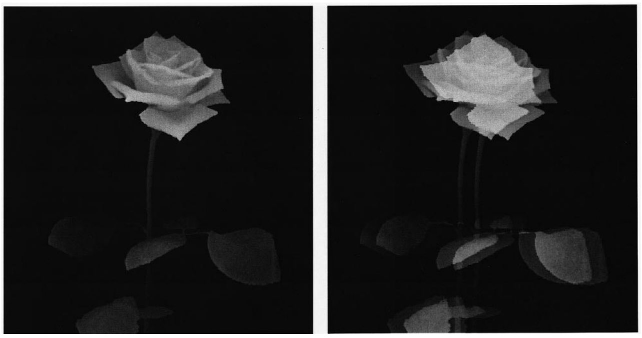

FIG. 2. Disparity map of Fig. 1. The depth of the object is indicated by brightness. (a) (left): Disparity map in 3D of Fig. 1(a). (b) (left): Demonstration of the perceived 3D structure in the stereoscopic state $s=2$ in Fig. 1(b) obtained after looking at Fig. 1(a) first. This is not a used disparity map, and serves only for demonstration purposes. The disparity map for all autostereograms was the map shown in (a).

maps of the objects. Such a disparity map is shown in Fig. 2(a), which is the solution of Fig. 1(a). The depth of each part of the 3D structure is encoded in the gray values of the assigned pixel of the depth map as a byte field. The background is black with the gray value of 0 and the foreground is white with the gray value of 256.

Autostereograms have to be viewed in a special stereoscopic way. The fixation of the two eyes has to decouple: each eye has to fixate on horizontally different location on the paper plane. The horizontal distance Δ of these fixation locations determines the spatial perception of the autostereogram. Autostereograms have a repetitive geometry of vertical stripes with a horizontal period length p. Therefore the following relation holds

$$\Delta = sp, \tag{1}$$

with $s=\ldots,-2,-1,0,1,2,3,\ldots$ denoting multiples of the horizontal period length. Therefore 3D perception becomes extremely ambiguous. In general, two different ways of stereoscopic viewing of autostereograms exist:

(i) Uncrossing the eyes $(s<0)$. The left eye focuses on a pattern on the left and the right eye on a pattern more to the right. This more relaxing viewing technique enforces a three-dimensional perception *behind* the paper plane.

(ii) Crossing the eyes $(s>0)$. Here the left eye focuses on a pattern to the right and the right eye focuses on a pattern more to the left. Therefore a three-dimensional object is perceived *in front* of the paper.

II. CONTROL OF AUTOSTEREOGRAM VISION

A. Classification of different 3D percepts

During the initial phase of viewing an autostereogram, the trivial perception of a flat, meaningless surface of random dots is dominant. This visual state s may be classified by $s=0$. After some further observation time, varying from person to person (from seconds to years), a transition of perception occurs and a different 3D aspect can be seen.

Looking at Fig. 1(a) the most probable 3D aspect is the perception of a single 3D rose, as shown in Fig. 2(a). To get this perception, our visual system uncrosses the eyes in such a way that the horizontal shift from the left image to the right image is approximately one period length p of the repetitive pattern. The background depth is even exactly one period length. This stereoscopic perceptual state can be classified by $s=1$.

Different 3D aspects can be seen by uncrossing the eyes even more. The 3D background impression becomes deeper and in the foreground different 3D ghost images become apparent. This is caused by stereoscopic fusion of distances Δ on the screen of two or more period lengths, which is denoted by the perceptual states $s=2,3,\ldots$. The case $s=2$ can be studied in a convenient way by looking at Fig. 1(b) after looking at Fig. 1(a) first. Figure 1(b) is designed with half of the period length of Fig. 1(a), which automatically leads to the perception $s=2$. The appearing 3D structure is similar to the pattern depicted in Fig. 2(b). It shows two identical 3D roses intersecting with each other spatially, which produces the impression of ghost pictures. While the trunks do not interact with each other, in the intersection of the two roses a ghost picture of a third rose can be seen in the middle foreground. On the other hand, if you look at Fig. 1(b) without looking at Fig. 1(a) first, you may still see the single rose $s=1$, which demonstrates the ambiguity of Fig. 1(b). The crossed eyes cases are classified identically by negative values of $s=-1,-2,\ldots$. The effect of ambiguity and the occurrence of ghost pictures for visual states with $|s|\geqslant 2$ was simulated recently by means of a synergetic computer [7].

B. Control parameters of 3D perception

Here we ask the following questions: Which factors of the experimental design can clarify which ambiguous spatial states s and how? Can the process of the perception of spatial states s be influenced and controlled from the outside? And which of these so-called control parameters causes switching among percepts? In our experiments, the impacts of different control parameters on the dynamical process of depth perception of subjects are studied as well as properties of the induced phase transitions in their perception. We show that the bias of the percepts s can be changed by changing the design of the autostereograms. There is a variety of possible candidates influencing the stereoscopic correspondence problem and the perception of autostreograms. Possible control parameters are the brightness, the tilt, the location on the visual field, the amount of structure, or the intensity of an additional color pattern. All these factors can potentially either disturb or enforce different states s of depth perception. In our experiments, the influence of probably the two most important control parameters of stereoscopic perception will be studied, namely, period length p and noise strength Q.

1. Horizontal period length p of the repetitive stripes

The period length p of the repetitive patterns in pixels is expected to be an ideal parameter to control 3D perception in autostereograms, because it directly influences the vergence angle ϕ of the two eyes. The lines of sight of the two eyes for a plane 3D background pattern intersect with the screen plane at anchoring points with a horizontal assignment distance of $\Delta = ps$. For a subject in a viewing distance d to the screen and a interpupillary distance d_e, the following geometric relation holds:

$$\phi = 2\ \arctan\left(\frac{d_e - ps}{2d}\right). \tag{2}$$

The smaller the period length p is chosen, the larger is the vergence angle ϕ, and vice versa. The lower limit for the vergence angle ϕ is given by physical restrictions of the geometry of the autostereogram. The upper limit of ϕ is determined by viewing the screen surface $(s=0)$ with $\phi_u = 2\ \arctan(d_e/2d)$. It is a reasonable guess that there is a favored parameter range of vergence angle ϕ between these limits for every subject. The more the actual vergence angle is different from this range, the more uncomfortable are the viewing conditions for the subjects. If this is true, the 3D percepts s are biased and influenced by the period length p, and the perception of the subjects may be changed to different percepts s. An increase of p therefore should favor a decrease of s and vice versa and p should serve as a control parameter of depth perception.

2. Randomness of the surface structure

Random noise may be added to the autostereogram pattern in the form of a random mask. In our experimental setup, the strength of noise Q could be changed from $Q=0$ (no noise) to $Q=1$ (maximum noise). A noise strength of $Q=0.5$ means that 50% of the pixels of the autostereogram pattern are assigned to a random gray/color value. By adding increasing random noise on the autostereogram, the visibility of the stereoscopic structure decreases. As preliminary experiments showed, the 3D visibility is also dependent on the perceptual state s. This means that the higher the perceptual state s, the better the visibility of the 3D structure. It is reasonable to assume that there is a minimum individual visibility necessary for the 3D recognition for each subject. As soon as this threshold value is reached, the percept s should become very difficult or impossible to recognize. Therefore the depth perception is biased, which leads to higher states of s with a better 3D visibility. This means that the strength of random noise should serve as a control parameter of depth perception.

C. Experimental methods

The autostereograms were presented to the subjects on a computer screen with a horizontal resolution of 1000 pixels. The display was selected as black and white. The distance d of the subjects to the screen and interpupillary distances d_e were measured. In the course of the experiment, two different objects were used: the rose as shown in Fig. 1 and a configuration of two rabbits. The impacts of the two control parameters period length p and the strength of noise Q on the 3D perception of subjects were studied in four different experiments. The parameter values of the period lengths of the repetitive stripes were changed from a minimum $p=6$ pixels to a maximum $p=566$ pixels and the values of noise strength Q were changed from $Q=0$ to $Q=0.4$. The values of the control parameters were changed in the course of time. Each autostereogram was displayed for a given duration on the screen, which was controlled by a computer program. The spatial perceptions s of the subjects were recorded. Therefore the subjects had to press different buttons of the computer mouse with their right index finger.

D. Subjects

Subjects included four people (two male, two female) with a proven ability to detect 3D structure in autostereograms. The interpupillary distances of the subjects were measured. The subjects were placed at a comfortable viewing distance from the computer screen. Subjects 1, 2, and 4 were seated 71 cm from the screen, subject 3 was seated at a distance of 56 cm.

III. EXPERIMENT 1: REACTION TIMES WITHOUT NOISE

A. Motivation

Is the recognition of a special percept s enforced by presenting an autostereogram with a special period length p? What is the relation between the period length p and the reaction times of the subjects to recognize a 3D percept? And does this reaction time depend on the individually perceived state of depth s? If the answer is yes, reaction times can be seen as a measure for the bias induced by the period lengths to the different perceptual alternatives s. To answer these questions in this experiment, the reaction times of the subjects watching autostereograms with different period lengths p were recorded and studied.

B. Methods and procedure

Two different kinds of autostereograms were presented to the subjects in two different sets of trials. In the first set, the 3D stimulus presented consisted of two rabbits. In the second set, the stimulus object was a rose, as in Fig. 1. Each set consisted of four identical trials. In each of the trials, 50 patterns with randomly changing period lengths p, and therefore different stereoscopic depths, were presented to the subjects. As soon as a stereoscopic perception $s \neq 0$ occurred, the subjects had to indicate it by pressing the left mouse button with their right index finger. The reaction times were measured and recorded. The subjects had to identify their perceived spatial state s by pressing the mouse button a second time. Pressing the left mouse button indicates the simple depth perception $s=1$, the middle mouse button indicates $s=2$, and the right mouse button indicates higher states $s \geqslant 3$. The maximum presentation time of each autostereogram was limited to 15 sec. In the short pause between the presentation of two autostereograms, a neutral 2D masking picture was presented and the subjects were asked to concentrate on the screen surface. The period lengths were in the range of $6 \leqslant p \leqslant 406$ in an exponentially weighted distribution. Therefore more of the interesting smaller values of p were presented. Another reason was to avoid frequent occurrences of patterns with very high period lengths, which were very tiring for the subjects. The random sequence of p was produced by a random number generator and stored on a file. This file was presented during each set of the experiment, twice in a forward direction and twice in a backward direction, and it was the same for each subject.

C. Results

We took the average over the four trials for the two stimuli and for each of the subjects. The averaging procedure

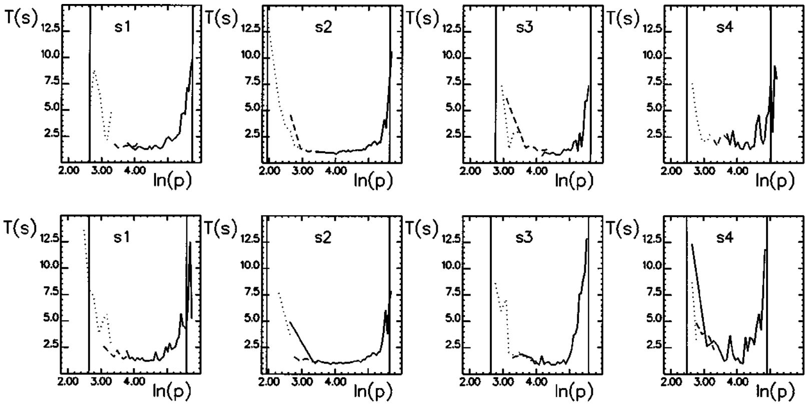

FIG. 3. Averaged reaction times in seconds over four trials for four subjects ($s1$, subject 1; $s2$, subject 2; $s3$, subject 3; $s4$, subject 4) and two different autostereogram patterns. Upper panel: rabbits pattern; lower panel: rose pattern. The reaction times are depicted versus the natural logarithm of the period length p of the presented autostereogram in pixels. The solid line represents the $s=1$ perceptual state, the dashed line the state $s=2$, and the dotted line the state $s\geqslant 3$. A curve element of the percept s is only dotted if in at least two of the trials the same percept s was identified. The vertical bars indicate the limits of stereoscopic perception.

was done as follows. For each value of the period length four different reaction times were recorded. They were averaged individually for different spatial states s, if there were at least two results for the same state s. The results for the four subjects and the two stimuli are shown in Fig. 3.

The measured reaction times are depicted versus the period lengths p on a logarithmic scale. It can be clearly seen that the behavior of the measured reaction times was independent of stimulus type. All the subjects had reaction time curves in a typical u-shape from when plotted versus p. A minimum reaction time T_{min} is reached in an intermediate area between $p=60$ and $p=140$ pixels. The reaction times vary from subject to subject, and they are an indicator of the skill of stereoscopic perception as already known from studies with random dot stereograms, e.g., [8]. The results for the four subjects $(s1,s2,s3,s4)$ and the two stimuli (stimulus 1: rabbit pattern, stimulus 2: rose pattern) can be seen in Table I. The average minimum reaction times T_{min} and the average reaction times $\langle T\rangle$ in seconds are presented, as well as their standard deviations (SD) in seconds. T_{min} is averaged over the four trials and $\langle T\rangle$ is averaged over the 50 stimuli of each of the four trials.

The results of the reaction times are discriminated in Fig. 3 in different curves for different percepts s. The solid line indicates the reaction times of the simple stereoscopic perception $s=1$, which was observed in the widest parameter range of period lengths. For higher period lengths p, the reaction times increased and at some point no depth could be perceived anymore. The transition points of the period lengths in pixels of fading 3D perception are presented in Table II for the four subjects and the two stimuli.

In Fig. 3, the transition from the perception $s=1$ to no depth perception $s=0$ is indicated by the vertical bar at the high end of the period length spectrum. The bar indicates the lowest period length p where the subject in two or more of the trials was in the state $s=0$. At the lower end of the period length range, other perceptual states were dominant. This can be seen by the existence of the dashed line, which indicates the perceptual state $s=2$. At the intersection I_{21}, between the solid and the dotted lines, the autostereogram becomes ambiguous. This interaction is located on different period lengths for the different subjects again. For even smaller values of p another intersection I_{32} to the perceptual states $s\geqslant 3$ appears, followed by another threshold behavior: the three-dimensional resolution and information gets lost and the reaction times increase until the object becomes invisible. This transition point is indicated by the vertical bar at the high end of the period length spectrum.

D. Discussion

The results of Experiment 1 clearly demonstrate that there is an ambiguous 3D perception in all subjects. The recorded reaction times are dependent on the size of the presented period lengths p and serve as a measure for the bias of the individual perceptual alternatives s. As Fig. 3 shows, the period length p serves as a control parameter for the percep-

TABLE I. Average minimum reaction times T_{min} and average reaction times $\langle T\rangle$ in seconds and their standard deviations (SD) for the four subjects $(s1,s2,s3,s4)$ and two stimuli (Stimulus 1, rabbits pattern; Stimulus 2, rose pattern).

Subject	Stimulus 1		Stimulus 2		Stimulus 1		Stimulus 2	
	T_{min}	SD	T_{min}	SD	$\langle T\rangle$	SD	$\langle T\rangle$	SD
$s1$	1.12	0.13	0.92	0.18	5.21	0.78	5.14	0.44
$s2$	0.78	0.02	0.84	0.08	4.07	0.16	4.19	0.46
$s3$	0.70	0.03	0.69	0.05	5.59	0.53	6.24	0.38
$s4$	1.08	0.09	0.83	0.11	8.36	1.43	9.32	0.80

TABLE II. Transition points of period length in pixels indicating disappearing 3D perception for the four subjects $(s2,s2,s3,s4)$ and two stimuli (Stimulus 1, rabbit pattern; Stimulus 2, rose pattern).

Subject	Transition (pixel)	Transition (pixel)
	Stimulus 1	Stimulus 2
$s1$	304	304
$s2$	353	357
$s3$	353	340
$s4$	243	212

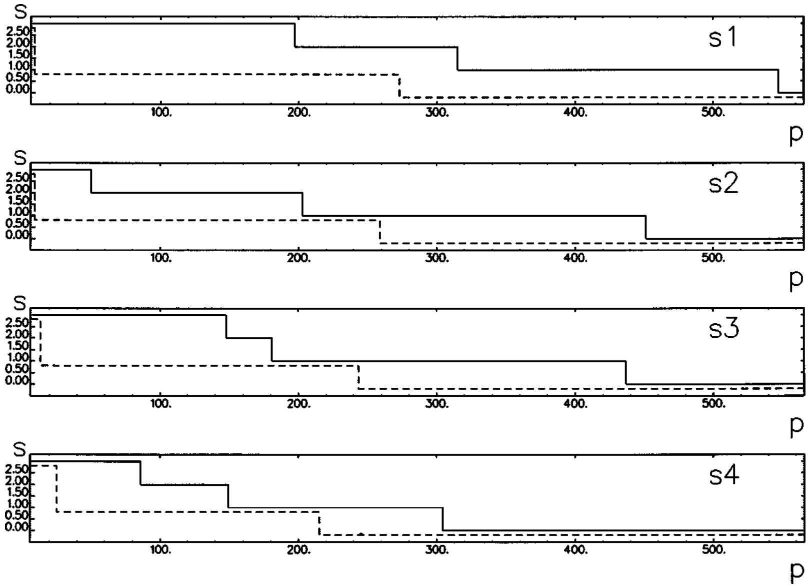

FIG. 4. Hysteresis curves of the four subjects, averaged over four runs. The states of spatial perception $s(t)$ are depicted versus the period length p in pixels. The cases of increasing period length are indicated by the solid lines; decreasing period length cases are indicated by the dashed lines. A remarkable hysteresis effect is visible. In the case of increasing p, a cascade of perceptual transitions occurs. In the case of decreasing p there is only one transition visible, because the hysteresis of the state $s=1$ is so strong that no higher states s occurred.

tion of the different spatial alternatives s. Higher values of p bias 3D perception to smaller values of s and vice versa. This relation opens up further experiments, such as the study of hysteresis in Experiment 2. In Fig. 3, each percept s has a typical u-shape reaction time curve with a different minimum at the favored disparity range of each subject. The intersections between the individual reaction time curves determine the parameter ranges p of unbiased ambiguity between the percepts. This knowledge about the ranges of ambiguity for each subject is important for the design of the more complicated experimental architecture of the stochastic resonance effect in Experiment 4.

IV. EXPERIMENT 2: PHASE TRANSITIONS AND HYSTERESIS

A. Motivation

Fender and Julesz [9] found a large amount of hystersis in the perception of random dot stereograms. Is this hysteresis also present in the perception of autostereograms? Does the hysteresis depend on the size of the horizontal period length p of an autostereogram? And can changes in the size of p induce phase transitions or even a cascade of phase transitions in the 3D perception of the subjects? To decide these questions in detail, we designed an experiment showing autostereograms with continuously changing period lengths p.

B. Methods and procedure

In our experiment, the period length p is changed by a continuous increase or decrease. Preliminary measurements showed a very strong effect of hysteresis. Therefore the parameter range for the period length p was extended to $6 \leqslant p \leqslant 566$ pixels. The experimental setup was as follows: The rabbit stimulus was presented to the same four subjects on another day with the same viewing distance and illumination background. The experiment was divided in two different trials. In each trial, the period length p was first increased step by step from $p=6$ to $p=566$ pixels in an exponential scale with 100 different values of p. Then, in the opposite way, the period lengths were diminished again from $p=566$ back to $p=6$. This cycle was repeated four times in both trials. In the first trial, the pictures were presented as continuously as possible, only restricted by a delay time of 0.6 sec because of computer speed. In the second trial, the pictures were shown as a slide show with an additional pause of 1.0 sec for each picture. The results for the two trials were averaged over the four cycles. The subjects reported each change of their spatial perception by pressing the buttons of the mouse. They indicated their new states of spatial perception by $s=1$, left mouse button; $s=2$, middle mouse button; $s=3$ or $s=0$, right mouse button.

C. Results

The average over the four cycles was taken for each subject and trial. The results are shown in Fig. 4 for the four different subjects in the continuous trial. The results of the second trial with the paused presentation were very similar and are not presented. In Fig. 4, the development of 3D percepts $s(t)$ is plotted for each subject as a function of period length in pixels. The solid line describes the perceptual changes as a function of an increase in the control parameter period length, the dashed line of a decrease in period length. For each of the subjects, a cascade of perceptual switches from $s=3$ to $s=2$ to $s=1$ to no depth perception $(s=0)$ can be seen in the solid curves for increasing period lengths. These switches take place in a striking and sudden change of spatial perception. The dashed curves show the development of spatial perception in the case of decreasing period lengths. Starting at the highest period length of 566 pixels the perception stays constant at $s=0$ until transitions to $s=1$ in the range 210–270 pixels take place. Notice the difference of this transition points to the opposite transitions

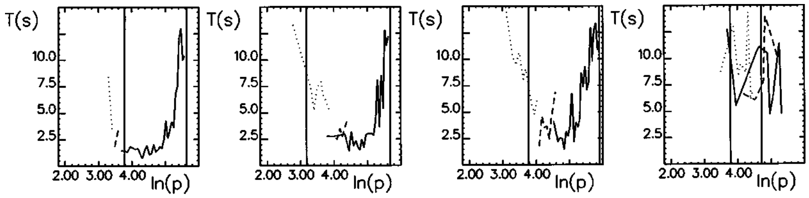

FIG. 5. The impact of noise to the perception times for one typical subject $(s1)$ viewing the rose pattern. As in Fig. 3, the reaction times, in seconds, are plotted versus the period length p in logarithmic scale. The solid line represents the state $s=1$, the dashed line $s=2$, and the dotted line $s\geqslant 3$. The vertical bars again indicate the limits of 3D vision. The strength of noise Q is increased from the left to the right figure from $Q=0.1$ to 0.2 to 0.3 and to 0.4. As can be seen, the results are shifted to higher period lengths.

indicated by the solid lines from $s=1$ to $s=0$, which are in the range 305–545 pixels. This shows the persistence of a given percept over a large parameter range is dependent on the direction of period length change. This remarkable hysteresis effect can also be seen in the fact that none of the subjects showed a transition from the state $s=1$ to $s=2$ when the period length was decreased further. This can be seen in the lack of the states $s=2$ for the dashed lines. The state $s=1$ was stable for all the subjects until very small sizes of p. Then the resolution gets too small and no spatial perception is possible any more, which is denoted by $s=3$.

D. Discussion

The results of our experiment show that the period length p of an autostereogram serves as a control parameter of 3D perception. By changing the size of the period length, the different percepts s are biased, which enables and controls impressive phase transitions of perception. These phase transitions show a remarkable effect of hysteresis. Therefore the parameter range of the period length had to be extended from the maximum size of $p=406$ pixels in Experiment 1 to $p=566$ in Experiment 2. The strong hysteresis effect helped subjects to retain their stereoscopic perception $s=1$ until extreme period lengths and viewing angles by increasing p. The knowledge of the amount of hysteresis in certain parameter ranges for each subject will be helpful for the design of Experiment 4 to study properties of stochastic resonance.

V. EXPERIMENT 3: REACTION TIMES WITH NOISE

A. Motivation

What is the role of noise in the depth perception of autostereograms? Can noise play a helpful role in 3D perception or is it only disturbing the visibility of the pattern? Can noise serve as a control parameter of perception? And, if yes, how does it interact with the control parameter of period length? These questions are studied in Experiment 3. Noisy autostereograms with different period lengths and different strengths of noise are presented to subjects and their reaction times are recorded.

B. Methods and procedure

The experimental setup is the same as in Experiment 1. The same set of patterns with the same random period length are presented with additional statistical noise on the patterns. The same four subjects were asked to indicate the onset of their 3D perception by clicking the left mouse button. The 3D stimulus was the rose. At the onset of 3D perception, the subjects had to press the left mouse button and the reaction time was recorded. The subjects indicated their states s of perception as usual ($s=1$; left mousebutton; $s=2$, middle button; $s\geqslant 3$, right button). The maximum presentation time of each autostereogram was again limited to 15 sec. The experiment consisted of four trials with different strengths of noise. The noise was increased in equidistant steps from 10 to 40 % of noise. A noise level of 10 % means that 10 % of the dots of the autostereogram are determined by a random distribution.

C. Results

The results for a typical subject $(s1)$ are shown in Fig. 5. The reaction times are plotted versus the period length p in logarithmic scale. The different perceptual states s are indicated by the different curves (solid, $s=1$; dashed, $s=2$; dotted, $s\geqslant 3$). The four trials with different strengths of noise Q are depicted in the four different figures. Noise strength increases from the left $(Q=0.1)$ to the right figure $(Q=0.4)$. The vertical bars again indicate the limits of 3D perception with $s\neq 0$. As can be seen, the curves of the reaction times still show a u-shape behavior dependent from the period length. With increasing noise, this relation is more and more disturbed because of the low remaining visibility of the patterns. Therefore in the case $Q=0.4$, the u-shape form is no more clearly visible. The limit of visibility for high noise strengths can be seen in the decreasing number of recognized 3D percepts. Subject 4, for example, was only able to perceive 3 of the 50 patterns in the case $Q=0.4$. As can be seen from Fig. 5, the general perceptual behavior of the subject in the cases $Q=0.1$, 0.2, 0.3 was similar to the behavior in the case of no noise. The subject was still able to clearly discriminate the different percepts. The percepts $s=1$ appeared at high values of the period length, the percepts $s=2$ at intermediate values and the percepts $s=3$ at low values.

There are several drastic impacts of noise changing the properties of reaction times. As can be seen in Fig. 5, the average reaction times $\langle T\rangle$ and the minimum reaction times $T_{\min}$ increase with noise. This is clearly visible in Fig. 6 for all subjects.

In the left part of Fig. 6, the minima of the reaction times are plotted in seconds versus the strength of noise Q. The case without noise is also included by using the averaged results for the four trials with the rose from Experiment 1. All the subjects show a clear increase in their reaction times. In the right part, the average reaction times are plotted versus the strength of noise Q. Again, the general trend of increasing reaction times with increasing noise is visible, although

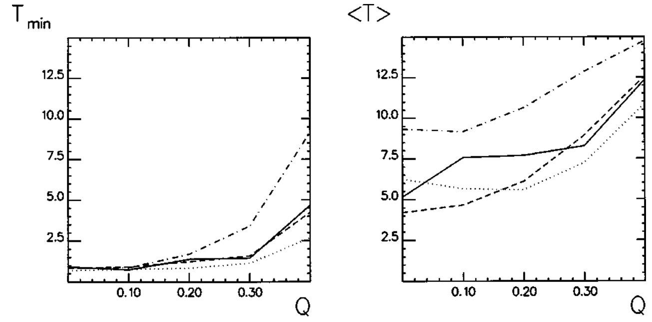

FIG. 6. Minimum reaction times and averaged reaction times, in seconds, averaged over a trial versus noise strength Q for all the subjects. The solid curve represents subject $s1$, the dashed curve subject $s2$, the dotted curve subject $s3$, and the dashed-dotted curve subject $s4$.

in some special cases noise improved perception slightly, e.g., for subject $s3$ from $Q=0$ to $Q=0.2$ or for Subject 4 from $Q=0$ to $Q=0.1$.

Another strong impact of noise to the reaction times is visible in the dependency of reaction times from the period length. As can be seen in Fig. 5, the reaction time curves are shifted to higher values of period length with increasing noise. This general shift can be clearly seen in the shift of the positions of the minima, the locations of the vertical bars, and the locations of the intersections of the parameter ranges of different percepts s. This general shift to higher values of period lengths can be better seen in Fig. 7.

The locations of the intersection points between the curves of the different states s are plotted in pixels versus the strength of noise Q for all of the four subjects. Four threshold values for the ambiguous intersections of 3D percept curves are calculated. This leads to four different curves: I_{03} (intersection between $s=0$ and $s=3$), I_{32} (intersection between $s=3$ and $s=2$), I_{21} (intersection between $s=2$ and $s=1$), and I_{10} (intersection between $s=1$ and $s=0$). The relation

$$I_{03} \leqslant I_{32} \leqslant I_{21} \leqslant I_{10} \tag{3}$$

holds by definition. The case without noise is again included by using the averaged results for the four trials with the rose from Experiment 1. As can be seen in Fig. 7, the locations of the intersections I_{03}, I_{32}, and I_{21} increase with increasing noise for all the subjects with the exception of Subject 3 in the high noise ($Q=0.4$) case. This means that the external physical noise is pushing spatial perception to states s of a higher level. Only the behavior of the curve I_{10} is different. In most of the cases, increasing noise leads to a decrease of I_{10}. Therefore the whole parameter range of p for identified 3D patterns in general diminishes with increasing strength of noise, as well as the total number of perceived autostereograms.

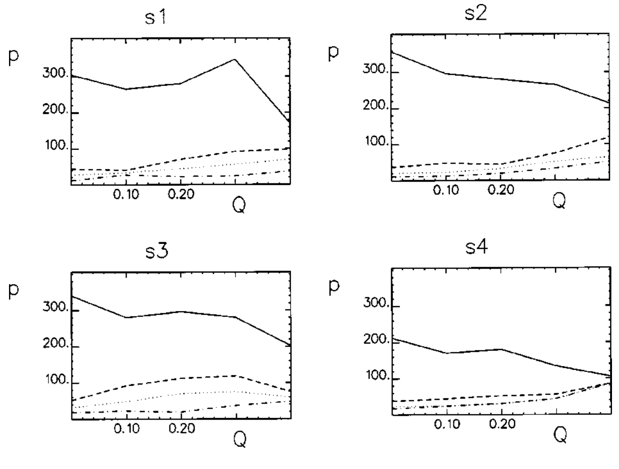

FIG. 7. Period lengths p of the points of ambiguous intersections between two states in pixels versus the strength of noise Q for the four subjects $s1$, $s2$, $s3$, $s4$. The dashed-dotted curve I_{03} represents the transition points between $s=0$ and $s=3$, the dashed curve I_{32} the ambiguous cases between $s=3$ and $s=2$, and the dotted curve I_{21} the transition points between $s=2$ and $s=1$. All these curves increase with Q, with the exception of the somehow unreliable 40% state of noise. The solid curve I_{10}, which represents the transition points between $s=1$ and $s=0$, usually decreases with noise. But for some cases it also increases with noise, which shows the helpful role of noise for depth perception.

But obviously I_{10} does not always decrease. Sometimes the curve I_{10} increases with increasing noise and does not necessarily have its maximum in the case without noise. This can be clearly seen in the results of Subject 1, which had its maximum value of I_{10} at $Q=0.3$ and in parts of the curves of Subjects 3 and 4. This demonstrates again that physical noise plays a helpful role in some cases and enables subjects to perceive 3D structures, which they were unable to perceive in the no-noise case.

D. Discussion

Experiment 3 shows that random noise not only diminishes the visibility of autostereograms. It influences and biases the 3D percepts s and can even improve visibility in some cases. Therefore the physical noise serves as a second control parameter of perception. We showed in our experiment that increasing noise forces 3D perception to higher perceptual states s. This driving force increases with higher noise strengths until a maximum noise level is reached. For all the subjects at the noise level of 40 % this visibility limit was reached, but still the driving effect of the noise is visible. The dependency between the control parameters of noise strength Q and period length p can be used to demonstrate the effect of stochastic resonance in 3D perception.

VI. EXPERIMENT 4: STOCHASTIC RESONANCE

A. Motivation

The recently found effect of stochastic resonance [10–14] can be studied in a visual experiment with autostereograms. Previous experiments with noise enhanced visual perception in humans are discussed in [15–17]. Noise enhanced perception in an animal (in the form of an electrical sense) is discussed in [18]. Dynamical systems show the behavior of stochastic resonance with the following special ingredients:

(i) An external periodic force has to be present with a certain frequency and amplitude. This amplitude is less than a critical value, which would enforce the whole dynamical system to oscillate. This means that the external periodic force is not strong enough to lead the system to oscillations.

(ii) Additive noise with a certain strength has to be present. This noise has very different impacts on the dynamical system, which depend on the size of its strength. For small strengths of noise there is no big impact to the system, which still remains stable. With increasing values of fluctuations, the system becomes more and more unstable. In an intermediate range of noise strength, the effect of stochastic resonance occurs: the system oscillates at a special frequency connected to the external periodic force. This means that the disordered fluctuations are the reason for an ordered oscillating state. With higher strengths of noise, this ordered state created by noise is destroyed by itself again. Can these effects be seen in autostereograms? Is stochastic resonance a feature of the dynamics of depth perception? We will show that our experiment is a very good test for these questions, if appropriate assignments of the experimental design are made.

B. Methods and procedure

The experimental setup is an extension of Experiment 2 but it includes noise. The multistable dynamical system supposed to show the stochastic resonance is the perceptual system of the subjects with the percepts $s(t)$. The external driving force is represented by an oscillation of the period length p with a certain frequency ω_0 and a very small amplitude A. The parameter range of these oscillations was chosen in the given range of ambiguity between the stereoscopic states $s=1$ and $s=2$, which were determined in Experiments 1 and 2. The average of the oscillations of the period lengths over time is set to the value of the ambiguous intersection I_{21} for each subject individually. The fluctuations, which are crucial for the stochastic resonance effect, are represented by changing noise added to the picture.

For this experiment only the two most skilled subjects were selected. The 3D rose stimulus was presented in five trials to the subjects. The ambiguous period lengths I_{21} were selected for both of the subjects individually ($s1$, $I_{21}=45$ pixels; $s2$, $I_{21}=30$ pixels), as well as the amplitudes of the oscillations of the period lengths. The amplitudes were chosen as $A=30$ pixels for Subject $s1$ and $A=20$ pixels for Subject $s2$, which was small enough not to enforce phase transitions of s. In eight cycles with the resolution of 25 different period lengths per half cycle the autostereogram was continuously shown to the subjects. In each of the five trials a different average strength of noise—which stands for the inverse visibility of the pattern—was added to the autostereogram. In order to establish fluctuations into the system, the control parameter of visibility has to be noisy. Therefore in the five trials, autostereograms with average visibilities $\langle Q \rangle = 0$, 0.1, 0.2, 0.3, and 0.4 were presented with noise on the visibilities of the strength $q=0$, 0.1, 0.2, 0.3, and 0.4. The subjects had to report every change of their perception by clicking a mouse button ($s=1$, left; $s=2$, middle; $s \geqslant 3$, right button). Additional oral reports were necessary with increasing total strength of the noise in order to discriminate higher states $s>3$ of depth perception. For a higher noise level the higher perceptual states became more dominant again as was already seen in the experiment before.

C. Results

The temporal evolution of the time series of the states $s(t)$ was calculated for each trial. In the first trial without noise, both of the subjects only saw one stable percept $s=1$ over all the time. The frequency of changes increased remarkably in the following trials. At a medium range of noise (20–30 %) oscillations with the same frequency as the weak external oscillation of the period length occurred. With higher noise (40 %), the average visibility was very weak. The perceptual states s increased and the transitions appeared in a more random way. This behavior can be clearly seen in Fig. 8, which shows the spectra of the time series of the perceptual states $s(t)$ for the different trials of the two subjects.

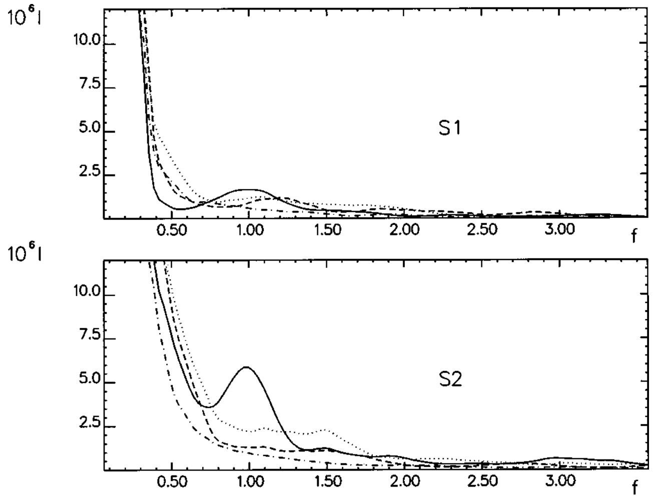

FIG. 8. Power spectrum of the curves of 3D perception $s(t)$ for different noise levels: 10–40% for the subjects $s1$ and $s2$ vs frequency. The frequency is normalized to the external force with the frequency ω_0. The coding of the curves is as follows: dash-dotted curve, 10% noise; dashed, 20%; solid, 30%; dotted, 40% noise. A peak of stochastic resonance can be seen for both subjects at the intermediate noise level of 30%.

The intensities $I(f)$ of the Fourier components of the time series $s(t)$ are plotted versus frequency f. The frequencies are normalized to the external driving frequency. The curves of the first trial are too small to be visible due to the constant state $s=1$ during the whole session. As can be seen, the intensities have a peak at the frequency of the weak external force $(f=1)$. The peak with the highest intensity and smallest width appears for the solid curve with intermediate noise level (30 %). This is a clear signature for the presence of stochastic resonance in the visual stereoscopic system.

This behavior becomes more visible in Fig. 9. For both of the subjects, a coefficient for the signal to noise ratio R of the different trials is calculated and plotted versus noise q. R can be defined as a quality factor [19,20] by

$$R=\omega_p \frac{h}{w}. \tag{4}$$

Thereby h is the maximum peak height of the spectrum, w is the width of this peak defined at the height of $(h/\sqrt{e})$, and ω_p is the frequency of the resonance peak. The smaller the relative width of the peak and the higher the peak, the more coherent is the oscillation of the perception. As can be seen, the maximum of R is located at an intermediate noise of $Q=0.3$ for both of the subjects.

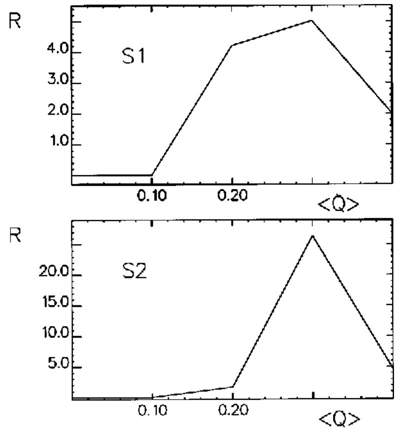

FIG. 9. Signal to noise ratio R as defined in the text vs noise for the subjects $s1$ and $s2$. A clear maximum in the trial with 30% noise is visible for both subjects.

VII. DISCUSSION

Autostereograms supply human depth perception with a new degree of freedom. Different three-dimensional perceptions can be seen in well-defined different levels of depth. Our visual system can choose from a variety of different alternatives, which is a general characteristic because stimulus patterns are always ambiguous and allow for two or more perceptual interpretations [21]. As can be seen in Experiments 1 and 2, the easiest stable state $s=1$ which is the state closest behind the screen, is usually perceived. But this is not always the case. For small period lengths p, all of the subjects reported higher spatial states $s \geqslant 2$ with a remarkable hysteresis effect. This leads us to the assumption that our perception instead looks for a comfortable viewing distance with a comfortable vergence angle.

As we showed in Experiment 3, the perceptual behavior drastically changed as soon as statistical noise was introduced to the patterns. The added random noise released the perception of the subjects into spatial states with a higher s. The key to this behavior is buried in the periodicity of the patterns of autostereograms. In the state of spatial perception $(s>0)$, the pixels in a certain horizontal distance $\Delta = ps$ are stereoscopically fused by assigning them perceptually to each other. This process becomes more difficult if the pattern is disturbed by a random noise of the strength Q.

In our experiments, we used only black and white pixels. A noise of the strength Q means in this case an actual noise strength of $(Q/2)$ on the screen, because half of the pixels on average had the same black/white state as the additional random mask. What happens if the autostereogram is stereoscopically fused $(s>0)$? Increasing perceptual states $s = 1,2,\ldots,s_{\max}$ lead to increasing assignment distances $\Delta = sp$. The maximum assignment distance $\Delta_{\max}$ for stereoscopic vision is limited by two conditions:

(i) The viewing angle $\phi_{\min}$ is limited by the fusion limit. In the extreme case $\phi_{\min}=0$, Eq. (1) leads to $\Delta_{\max}=d_e$.

(ii) Δ has to be equal or smaller than half of the total width b of the autostereogram, which leads to $\Delta_{\max}=(b/2)$.

Stereoscopic perception is induced by assigning the elements of the repetitive patterns to each other. Dependent on the size of Δ, the number n of repetitions of the periodic patterns change:

$$n = \frac{b}{\Delta} = \frac{b}{ps}. \tag{5}$$

Each of this Δ different horizontally assigned pixel chains with length n is expected to be useless or at least diminished in its use for depth perception as soon as it contains a noisy element. Under this hypothesis, that already one perturbed pixel in a horizontal chain of the length n is enough to destroy the stereoscopic fusion of the whole chain, it is obvious that the perceptual states with high $\Delta = sp$ and small chain lengths n appear less disturbed. Therefore the driving force of the noise strength Q in favor of higher perceptual states s can be understood. Noise brings the percepts s into a new order of viewing comfort. The higher the percept s, the sharper and less noisy appears the visibility of the 3D objects. Therefore the perception changes to higher percepts s.

This behavior can be described as a generalization of the Pulling effect, which is well known in the case of random dot patterns [4,22]. It was shown by Julesz and Chang [23] that the perception of depth can be changed by an additional mask with a different multiple period length. This mask appeared as noise in the former 3D state but vanished in the new deeper state. One difference of this pulling effect of noise in random dot pictures is that in our case the noise mask has no more periodicity at all. Therefore no stable perception with undisturbed visibility is possible for deeper states s either. Another difference and generalization of the Pulling effect between two states is the large cascade of spatially distinct states of depth s in our experiment.

We showed that noise is not only destructive. If there is a high perceptual pressure of noise, the eyes get inventive and are able to fuse higher period lengths p than they are without noise.

Noise has the function of destabilizing former stable states of perception. This function can be very helpful in some cases. For example, noise helps to outrun the strong hysteresis of spatial vision and makes the system more flexible. We showed this behavior by the detection of the property of stochastic resonance in the system. Because of hysteresis, small fluctuations do not affect the subjects' perception very much; therefore the perceptual state remains stable. But in an intermediate range of noise, the hysteresis was remarkably diminished and the perception of the subjects oscillated coherent to the oscillation frequency of the period lengths p between different perceptual states s. Larger amounts of the noise strengths destroyed the coherence of the oscillations of perception.

VIII. CONCLUSIONS

In our experiments we demonstrated the widespread impacts of two parameters on the stereoscopic perception of autostereograms: the period length p and the strength of additional noise Q. We showed that 3D perception can be influenced by both of the parameters and impressive phase transitions between different percepts take place. We showed the influence of memory by demonstrating the remarkable hysteresis effect. We showed that physical noise on an autostereogram not only diminishes the 3D visibility. In some cases noise was able to play a helpful role. This was demonstrated by the effect of stochastic resonance and by an improvement in reaction times and in the number of perceived test patterns in some cases. We also showed that physical noise is relative in spatial perception of autostereograms. Dependent on the individual percept s, the visibility of the patterns changed.

These alternative findings have to be compared with the predictions of already existing models of autostereogram vision [7] based on the synergetic computer [24,25]. The experimental results of the different properties of the controlled ambiguity of autostereograms have to be connected to models of stereoscopic vision. With special parameter values, as in the threshold curves in Fig. 7, for a well balanced ambiguity between different spatial states $s>0$, oscillations of perception between these states may occur. The higher the noise level, the less resistance of the hysteresis to the oscillations may be expected. This should lead to a higher oscillation frequency. The impact of stereoscopic vision on the properties of the multistability of perception has to be studied in full detail; for example, the impacts of the high hysteresis effect and noise on the speed of the perceptual oscillations. This will lead to a generalization of the synergetic model of depth perception in combination with the oscillating properties of a model of ambiguous pictures [26].

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