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A Dynamic Pattern Theory of Behavioral Change

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Intentional change of behavior is an essential phenomenon that theoretical biology cannot fail to address. Often, theoretical attempts to understand the problem and experimental study of behavioral change are quite unrelated to each other. Recent progress in formulating a strictly operational dynamic theory of behavioral patterns, however, offers a link between theory and experiment. Here the understanding of intentional change of behavioral pattern in this theoretical language is shown. The general formulation provides predictions on the relation between the dynamics of behavioral patterns and the nature of the process of behavioral change. Theoretically founded measures, including switching time and first exit time, are introduced that allow a characterization of this process. A concrete system involving temporally ordered behavior is modelled explictly on two experimentally accessible levels of observation. Switching time and first passage time measures are calculated from the theory and the results compared to recent experimental observations. We discuss the potential of the switching time measures for the more general study of behavioral patterns and their dynamics.

1. Introduction

A most distinctive feature of biological systems is its goal-directed or apparent purposiveness in the absence of direct environmental influence. This aspect of the behavior of higher organisms has been at the core of much philosophical (e.g. Sommerhoff, 1950; Granit, 1977) and theoretical (e.g. Rössler, 1974; Rosen, 1985) discussion. The theoretical problems associated with terms such as goal-seeking, purpose and intentionality are clearly relevant for theories of behavior. Unfortunately the connection of these concepts with experimental studies of behavior remains quite weak and somewhat elusive. Thus questions such as: What can be intended? Are intentions constrained by the dynamics of behavioral patterns? What is the nature of the process of intentional change of behavior? remain quite open. One reason for the apparent gap between theory and experiment may be the lack of a precise description of behavioral patterns and its explicit connection with mathematically formulated theoretical concepts.

Recent theoretical insights gained through the study of co-ordinated movement may offer a new pathway to understanding voluntary changes of behavior (Kelso & Schöner, 1987, 1988; Schöner & Kelso, 1988*a*,*b*). Based upon theories of self-organization and pattern formation in non-equilibrium systems, in particular synergetics (Haken, 1983), a theoretical language was developed to understand behavioral patterns. The most important aspect of this language is its operational nature,

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stressing a close theory-experiment relation. The central ideas are the characterization of behavioral patterns by collective variables, the determination of the dynamics of behavioral patterns and the study of their stability. The dynamic theory of behavioral patterns that emerged is applicable on several levels of observation (e.g. Kelso *et al.*, 1987; Schöner & Kelso, 1988*c*) and can address such questions as the influence of the environment and of memory on behavioral patterns (Schöner & Kelso, 1988*a,b*).

In this article we make a first step towards understanding intentional change of behavior within the dynamic pattern language. The goal of an intentional change of behavior is expressed as an intended behavioral pattern and is characterized by the same collective variables used to describe the behavior itself. We assume that this goal of intention can be identified, and we study its influence on the behavior by including the intended pattern in the behavioral dynamics. We propose new observables, in particular switching time and first exit and entry times, to characterize the process of intentional behavioral change and link it to the pattern dynamics. This linkage affords general predictions on the relation between pattern dynamics and behavioral change. Although the theory is formulated generally, a concrete realization of the theory in the form of a model of a specific system is also offered. Thus a theoretical analysis of recent experimental work (Kelso *et al.*, in press) on intentional change of patterns of movement co-ordination is provided on two levels of description. The measures of switching time and mean first passage time are found to be in qualitative and, within certain bounds, quantitative agreement with the experimental results.

The article is organized as follows: in section 2 we summarize briefly the central aspects of the dynamic theory of behavioral patterns including a discussion of the model system from movement co-ordination that initially motivated the approach. Next (section 3) we review generalizations of the theory that include the notion of behavioral information, in the sense of specific influences of the environment and memory on the dynamics of the behavioral patterns. In section 4 we present the dynamic theory of intentional change of behavioral pattern, including predictions about how intentional changes are influenced by the behavioral pattern dynamics. Measures defined in the theory that allow insights into the actual switching process are introduced in section 5. Sections 6 and 7 deal with our theoretical modelling of recent experimental work on intentional changes of rhythmic movement behavior. The collective variable dynamics are determined on the levels of the relative phase (section 6) and the component oscillators (section 7). We show how the two levels of observation are linked, by deriving the former from the latter (section 7). The measures introduced in section 5 are calculated and compared to experimental data. We summarize and draw some general conclusions in the final section.

2. Dynamic Theory of Behavioral Patterns

The main idea is to view behavioral patterns in terms of their dynamics. The key step is to characterize the behavioral patterns by collective variables. Thus the dynamics of the patterns can be determined as dynamics of these collective variables.

To be concise, we summarize the dynamic theory of behavioral patterns in the form of a set of theoretical propositions (Kelso & Schöner, 1987, 1988; Schöner & Kelso, 1988c) that will be discussed and illustrated below. (i) Behavioral patterns at a given level of observation are characterized by low-dimensional collective variables or *order parameters* whose dynamics are function-specific. (ii) Observable (i.e. reproducible, stationary over a certain time scale) behavioral patterns are mapped onto attractors of the order parameter dynamics. (iii) Biological boundary conditions (e.g. environmental context, task variables) act as parameters on the collective dynamics. These parameters may be quite unspecific to the resulting stable behavioral patterns, in which case we refer to the order parameter dynamics as *intrinsic dynamics*. (iv) Fluctuations determine various time scales (local and global relaxation times). Time scale relations account for observed metastability and switching among multiple behavioral patterns. Loss of stability leads to switching of behavioral pattern and the switching process is governed by stochastic order parameter dynamics. (v) If behavioral patterns are thus characterized at different levels of description, these levels may be related mathematically without introducing additional concepts.

Propositions (i) to (iii) may be viewed as a self-consistent loop through which a dynamic pattern description of a behavior may be obtained. Arriving at a set of collective variables and their dynamics through this loop then amounts to choosing a level of observation. We would like to stress the functionally specific nature of the behavioral pattern dynamics, which means that the dynamics are valid for a given biological function or task, independent of how this function is physiologically implemented. It is this functional pattern that is stable, as can be experimentally tested in each case (for a good example, see Kelso *et al.*, 1984). However, on other levels of observation, the physiological implementation of a behavioral pattern may itself appear as a functional pattern (Schöner & Kelso, 1988*c*). When we refer in proposition (iii) to these dynamics as *intrinsic*, we mean that, given the task, they are present in the absence of specific requirements defined by the environment, by memory or even intention (see sections 3 and 4). We do not wish to imply that such intrinsic dynamics are hard-wired constraints of the physiology.

For later reference we give a general mathematical formulation for the intrinsic dynamics. For a given level of observation, an order parameter x_i will be subject to

$$\dot{x}_i = f_{intr}(x_i, \text{noise}) \tag{1}$$

where f_{intr} is, in general, non-linear and is defined by its attractor structure. The intrinsic dynamics are assumed to depend on noise for two reasons: first, any system in an ordered state that is described by only a few collective variables, but contains many more degrees of freedom, is subject to the influence of these underlying high-dimensional dynamics (Haken, 1975). Second, as we will see below, noise plays a crucial role in determining various time scales that define the stabilities of the system. This feature becomes clear during the actual transient process when a system is changing state (Landauer, 1962; for recent review also Landauer, 1979).

The model system that motivated the formulation of the foregoing propositions involves co-ordinated rhythmic movements which represent a broad class of behavior in many species. The basic phenomenon is the following (Kelso, 1984): human subjects are asked to move two limbs rhythmically at a common frequency. Two patterns of co-ordination (in-phase: homologous muscles groups contracting together and anti-phase: homologous muscles contracting alternately) are found to be stably performed at various frequencies. However, when frequency of movement is increased, a spontaneous switch from the anti-phase to the in-phase pattern of co-ordination is observed, while the in-phase pattern remains stable at all reachable frequencies.

A dynamic pattern description can be obtained on one level of description by choosing the relative phase, ϕ between the rhythmically moving limbs as a collective variable. The observed phase-locked patterns are mapped onto point attractors of relative phase at $\phi = 0$ and $\phi = \pm\pi$. The simplest dynamics of ϕ that accommodates these (and also fulfills basic symmetry requirements) are (Haken *et al.*, 1985):

$$\dot{\phi} = -\frac{\mathrm{d}V}{\mathrm{d}\phi} \tag{2}$$

where $V(\phi) = -a\cos(\phi) - b\cos(2\phi)$. This model captures the observed phase diagram: for $0 < a < 4b$ two stable states $\phi = 0$ and $\phi = \pm\pi$ exist, while for $a > 4b > 0$ only $\phi = 0$ remains stable.

In such a theory the spontaneous switch from anti-phase to in-phase co-ordination is due to loss of stability (namely of the anti-phase pattern). When fluctuations are taken into account:

$$\dot{\phi} = -\frac{\mathrm{d}V}{\mathrm{d}\phi} + \sqrt{Q}\xi_t \tag{3}$$

(where ξ_t is gaussian white noise and Q the noise strength parameter) a number of predictions about the behavior near the transition are possible (Schöner *et al.*, 1986). Thus the validity of the theory can be tested experimentally, in particular by measuring stability and trying to detect the hypothesized loss of stability. For example, theory predicts that the variability of the anti-phase co-ordination pattern (but not the in-phase pattern) should increase as the transition is approached. Such *enhanced fluctuations* were observed experimentally in several studies (Kelso & Scholz, 1985; Kelso *et al.*, 1986). Another measure of stability, *relaxation time* (the time it takes to return to the co-ordination pattern after a small perturbation) was shown to increase upon approaching the transition. In experiments in which perturbations were mechanically induced during the co-ordinated movement, relaxation time was measured and such enhancement observed (Scholz *et al.*, 1987). In other experiments, in which the relaxation time was determined from the line width of the power spectrum of the relative phase fluctuations, the same enhancement was found (Kelso *et al.*, 1987).

A further aspect of the theory is that local and global relaxation times have to fulfill certain relationships for multiple patterns to coexist. In the experiments (Scholz *et al.*, 1987) it was indeed found that switching occurred as these relationships were violated. Finally, characteristic features of the actual transient switching process (like the mean duration of this process, the so-called mean *switching time*, and the distribution of the switching times) were predicted (Schöner *et al.*, 1986) and found experimentally to be in good agreement with theory (Scholz *et al.*, 1987).

In this experimental system another level of description, namely the level of the rhythmically moving components, is available. We may choose end effector position x and velocity $\dot{x}$ of a moving limb as collective variables—collective now with respect to underlying neuromuscular activities. The observed pattern can then be mapped onto a limit cycle attractor of the $(x, \dot{x})$ dynamics. The simplest model that captures experimentally observed amplitude-frequency-velocity relations is (Haken *et al.*, 1985; Kay *et al.*, 1987):

$$\ddot{x} + f(x, \dot{x}) = 0 \tag{4}$$

with $f(x, \dot{x}) = \alpha\dot{x} + \omega^2 x + \beta\dot{x}^3 + \gamma\dot{x}x^2$ where $\alpha < 0$, $\beta > 0$, $\gamma > 0$, $\omega > 0$ are constants. Experimental consequences of this description (stability, autonomy, stable frequency-amplitude relation, dimensionality) were tested (Kay *et al.*, 1987) with positive results. Thus we have a dynamic pattern description also on the component level. Haken *et al.* (1985) showed that a non-linear coupling between two such component systems leads to the observed relative phase dynamics. For example, from the coupling structure

$$\begin{aligned} \ddot{x}_1 + f(x_1, \dot{x}_1) &= g_{\mathrm{intr}}(x_1, \dot{x}_1; x_2, \dot{x}_2) \\ \ddot{x}_2 + f(x_2, \dot{x}_2) &= g_{\mathrm{intr}}(x_2, \dot{x}_2; x_1, \dot{x}_1) \end{aligned} \tag{5}$$

with $g_{\mathrm{intr}}(x_1, \dot{x}_1, x_2, \dot{x}_2) = k(\dot{x}_1 - \dot{x}_2) + l(\dot{x}_1 - \dot{x}_2)(x_1 - x_2)^2$ the dynamics of relative phase (eqn (2)) were derived in the limit $|\alpha| \ll \omega$. A significant feature of this formulation is that the experimentally observed phase transition emerges—because of nonlinear coupling—through a simple change of oscillation frequency.

3. Behavioral Information in the Dynamic Theory of Behavioral Patterns

The definition of a biological function or a required task does not always specify behavioral patterns completely. In some situations continuous adjustment of behavior to the environment may be required. As examples we may think of prey seeking or pursuit behaviors or, generally, movement in a structured physical environment. Similarly, behavioral patterns may be modified by learning in a way that is specific to what has been learned. Recently the dynamic theory of behavioral patterns was generalized to include these two types of influences on behavior (Schöner & Kelso, 1988*a*,*b*).

Environmental information is conceptualized as a *required behavioral pattern*. This information becomes meaningful to the biological system through formation of a pattern with the environment, a perception-action pattern. Similarly, memorized information is conceptualized as a behavioral pattern required by memory. This pattern has been established through learning and has thus become meaningful. We define information here as information with meaning, not be be confused with formal concepts of information without meaning (cf. Brillouin, 1962, section 20).

To fix ideas, we refer to such specific and meaningful information as *behavioral* information. Note that behavioral information in this sense can be expressed in terms of the same collective variables that are used to characterize the behavioral patterns. How physical aspects of the environment become transformed into such meaningful information is a difficult and interesting problem, related obviously to a theory of perception (see, e.g. Gibson, 1950; see also Shimizu *et al.*, 1988, for an approach to this question via dynamics). Similarly the question of how information becomes meaningful during the process of learning is a central problem of learning theory.

The following assumptions define a dynamic pattern theory of such behavioral information (Schöner & Kelso, 1988*d*): (i) the order parameter dynamics in the absence of behavioral information (intrinsic dynamics) persist in the presence of such information; (ii) behavioral information perturbs the vector-field of the order parameter dynamics (eqn (1)), that is, it forms a part of the behavioral dynamics. This perturbation is defined such that in the absence of intrinsic dynamics the order parameter dynamics have attractors corresponding to the required behavioral patterns. A general mathematical formulation in the notation of eqn (1) reads:

$$\dot{\mathbf{x}}_t = \mathbf{f}_{\text{intr}} + c_{\text{info}} \mathbf{f}_{\text{info}}(\mathbf{x}_t, t) \tag{6}$$

where c_{info} is the strength of the perturbation $\mathbf{f}_{\text{info}}$. The latter function is defined such that in eqn (6) it attracts the collective variable $\mathbf{x}_t$ toward the required behavior. For illustration, consider the case where the required behavioral pattern is a fixed point, $\mathbf{x}_{\text{req}}$ of the collective variable dynamics. Locally around $\mathbf{x}_{\text{req}}$ we can then generally linearize $\mathbf{f}_{\text{info}}$ as ($c_{\text{info}} > 0$):

$$c_{\text{info}} \mathbf{f}_{\text{info}}(\mathbf{x}_t, t) = -c_{\text{info}}(\mathbf{x}_t - \mathbf{x}_{\text{req}}). \tag{7}$$

Although formulated quite generally, already a few predictions emerge from such a dynamic view of behavioral information. First, the presence of intrinsic dynamics in eqn (6) will be felt as a systematic deviation of the *actual* behavioral pattern from the *required* behavioral pattern. This deviation will be in the direction (in terms of the collective variable space) of the stable states of the intrinsic dynamics. Second, qualitative effects of behavioral information can be seen when the corresponding perturbation in eqn (6) breaks a symmetry of the intrinsic dynamics. Then asymmetric solutions may be stable, or various solutions that result from each other by applying the symmetry operation may differ in their stability. Finally, another important qualitative effect of behavioral information is the possibility of phase transitions in the order parameter dynamics as the behavioral information changes. Such phase transitions exist if the intrinsic dynamics (i.e. the limit $c_{\text{info}} \to 0$ of eqn (6)) are qualitatively different from the dynamics defined by the behavioral information (i.e. the limit $c_{\text{info}} \gg 1$ of eqn (6)). For example, multiple patterns may be intrinsically stable, while only one—the required pattern—may be stable for large c_{info}.

This particular theoretical development (Schöner & Kelso, 1988*a,b*) occurred in the context of two experimental systems (Tuller & Kelso, in press; Yamanishi *et al.*, 1980) that involved again co-ordinated, rhythmic movement by humans. In Tuller & Kelso's experiments a temporally structured environment was present in the form of two metronomes that paced the two index fingers at the same frequency. By changing the relative phase of the two metronomes, the environmental information, required relative phase, was varied. In Yamanishi *et al.*'s experiment, subjects practiced several such relative phasing conditions until a certain criterion level of performance was reached. Feedback methods were used to enhance learning. Then a given relative phasing pattern was elicited, using two metronomes, which were turned off for actual measurement purposes ("free running"). Thus, in this case, the required relative phase corresponds theoretically to what we call memorized behavioral information. In both experimental studies two robust findings emerged. First, the mean performed relative phasing deviated systematically from the required task such that it was closer to the nearest intrinsic pattern, that is, in-phase or anti-phase. Second, the variability of the performed phasing was minimal in the two intrinsic patterns and large at intermediate conditions. Both sets of results were captured by models that implemented the general theory (eqn (6)) both on the level of relative phase dynamics (Schöner & Kelso, 1988*a*) as well as on the level of the component oscillator dynamics (Schöner & Kelso, 1988*b*). A number of predictions emerged that can further serve to test the theory.

4. Intention as Behavioral Information

The basic idea of the present extension of the dynamic pattern approach is to consider intention as behavioral information in the sense defined above. We conceptualize *intentional information* as an intended behavioral pattern, i.e. the goal of the intentional act. Implementing dynamic theory requires two basic assumptions: (i) the intrinsic and, if present, environmentally specified or memorized dynamics persist; (ii) intentional information defines a perturbation of the order parameter dynamics that attracts the behavioral pattern toward the intended behavioral pattern. Mathematically, in the notation of eqn (1) we may write (in the absence of environmental or memorized information):

$$\dot{\mathbf{x}}_t = \mathbf{f}_{intr} + c_{intent}\mathbf{f}_{intent}(\mathbf{x}_t, t) \tag{8}$$

where the constant c_{intent} measures the strength of the influence of intention and $\mathbf{f}_{intent}$ is defined by its attractor structure.

Of course, up to this point the approach is purely formal. To give meaning to this formulation we have to address the question of which patterns can be intended. Essentially this means: which behavioral patterns can become attractors? This question may be quite difficult to answer in general. However, the dynamic theory of behavioral patterns (including behavioral information) leads us to distinguish the following possibilities.

First, one might assume that intrinsically stable patterns can be intended. As the intrinsically stable patterns are already attractors of the behavioral dynamics, this assumption amounts to merely changing the strength of this attraction by an intention, i.e. changing the stability of the intrinsic pattern that is intended. In this way *intentional switching from one intrinsic pattern to another* could be viewed as the manifestation of a perturbation of the behavioral dynamics by intentional information. A second possibility is that environmentally-specified patterns can be intended which amounts to modifying the strength of the perturbation defined by environmental information (eqn (6)) and relates to the interesting problem of how environmental information becomes meaningful to the biological system. A third possibility is that a memorized behavioral pattern can be intended, which means that the strength of a perturbation defined by memory is modified. The corresponding process is that of activation of memorized information. As the last two possibilities represent interesting problems in their own right, we concentrate in this article on the first view of intentional information in the context of intentional switching between intrinsically stable patterns.

The conceptualization of the goal of an intentional change of behavior as behavioral information means that we measure this goal in the same space of collective variables in which we measure the behavioral patterns themselves. This is because behavioral information defines an attractor in that space, and hence is a set in that space. As a result we can address only changes of behavior within the behavioral space defined by the collective variables. Factors unrelated to the behavioral dynamics, such as performance criteria, cost functions, etc. cannot be accounted for in the present theory, unless new collective variables are found that make these factors part of the behavioral pattern and accessible to a dynamic analysis.

In a general sense, an intentional change of behavior requires the existence of additional degrees of freedom beyond those accounted for in the intrinsic dynamics. For example, the onset of intention may introduce non-autonomy or may be viewed as the effect of another dynamical system. This process, namely that of the actual change of the behavioral dynamics in time, is again interesting in and of itself. A large field of experimental study is concerned with physiological indicators of preparation or intention to act (for reviews see, for example, Kots, 1977; Eccles, 1985). Here we will assume that through such underlying processes intentional information is activated, and will study its effects on the behavioral dynamics as defined by eqn (8). As a result, we cannot, of course, address questions that are connected to these underlying dynamics of intention itself.

Already in this limit and at this level of generality, two general predictions emerge. First, the intrinsic dynamics will influence the process of intentional switching among available patterns. How to assess this influence will be discussed in the next section by introducing measures of the switching process. Second, intention, here defined as part of the behavioral dynamics, can change the dynamic properties (for example, the stability) of the behavioral patterns. The last point is conceptually important to reconcile behavioral constraints—as expressed by a dynamic pattern equation—with the idea of intentional modification of behavior. The dynamics of the collective variables that we have identified as intrinsic in a given functional or task environment may, of course, themselves be viewed as the dynamic expression of an intentional behavior. In this sense the distinction between intrinsic and intentional dynamics is not unique, but a question of the behavioral change in which one is interested.

5. Measures of the Switching Process

The process of intentional switching between intrinsically stable patterns is necessarily a transient phenomenon and cannot be analyzed in terms of measures, such as variability or correlation functions, that have meaning only in stationary states. However, several measures that allow for the characterization of such transients exist and have been successfully used in the study of transients in spontaneous switching (Schöner *et al.*, 1986). Here we discuss switching time and first exit and entry times.

5.1. SWITCHING TIME

To be specific we shall assume the intrinsic dynamics of a collective variable $\mathbf{x}$, (eqn (1)) in the form

$$\dot{\mathbf{x}} = \mathbf{f}_{\text{intr}}(\mathbf{x}_t) + \mathbf{q} \cdot \xi_t. \tag{9}$$

Here we express the noise dependence of eqn (1) explicitly by a number of independent gaussian white noise processes, ξ_t that act as additive stochastic forces (with a constant matrix q) on the collective variables. This form of the noise dependence can be justified for systems in which many independent degrees of freedom are responsible for the noise on the collective variable level. The solution of eqn (9) is then a Markov process.

We will study intentional switching between two stable stationary states of the deterministic dynamics, $\mathbf{x}_1$ and $\mathbf{x}_2$ with

$$\mathbf{f}_{\text{intr}}(\mathbf{x}_1) = \mathbf{f}_{\text{intr}}(\mathbf{x}_2) = 0. \tag{10}$$

Each state has a local relaxation time τ_1 and τ_2 defined here simply as

$$\tau_i = -\lambda_i^{-1} > 0 \qquad (i = 1, 2) \tag{11}$$

where λ_i is the largest eigenvalue of the linear stability problem for $\mathbf{x}_i$. The stochastic system (9) has, in general, a stationary solution that is attained on a much larger time scale τ_{equ} (see Landauer, 1979, for a discussion of the different types of stability involved). When both local relaxation times are finite, this stationary solution will correspond to a process that is distributed over both attractors with probabilities governed by the dynamics along the paths connecting these two states. If we look at the system on an intermediate time scale τ_{obs} with $\tau_i \ll \tau_{\text{obs}} \ll \tau_{\text{equ}}$ $(i = 1, 2)$ then we find *local stationarity*. By that, we mean that the system has relaxed to a stationary process contained in one basin of attraction, thus being dependent on initial conditions only in so far as one basin of attraction was chosen. In a first approximation, mono-stable dynamics obtained from linearization around that stable state may be used to characterize the process in this condition of local stationarity (Schöner *et al.*, 1986). Importantly, all these different time scales are accessible to measurement (see, e.g. Kelso *et al.*, 1987).

Let us consider the case where the system is initially prepared in state $\mathbf{x}_1$ in the above sense of local stationarity. We assume that an intentional change of behavior from $\mathbf{x}_1$ to $\mathbf{x}_2$ occurs. Specifically, we will assume that at time t_0 the dynamics are abruptly changed to dynamics that include the intentional information:

$$\dot{\mathbf{x}} = \mathbf{f}_{\text{intr}}(\mathbf{x}_t) + \mathbf{q} \, . \, \xi_t + \theta(t - t_0) c_{\text{intent}} \mathbf{f}_{\text{intent}}(\mathbf{x}_t). \tag{12}$$

Here c_{intent} is the relative strength of the intentional contribution to the dynamics. We may choose $\mathbf{f}_{\text{intent}}$, for example, as

$$\mathbf{f}_{\text{intent}}(\mathbf{x}_t) = \mathbf{x}_2 - \mathbf{x}_t \tag{13}$$

which is the simplest, linear form of a perturbation that attracts the system toward $\mathbf{x}_2$. The function $\theta(t - t_0)$ is a Heaviside step function:

$$\theta(t) = \begin{cases} 0 & \text{for } t < 0; \\ 1 & \text{for } t > 0. \end{cases} \tag{14}$$

This instantaneous onset of the intentional dynamics must be viewed as a crude first approximation. Our emphasis, however, is on the change of the behavioral dynamics consequent to the onset of intention (see discussion in section 4).

To characterize the transient process for $t > t_0$ we use the time-dependent probability density $P(\mathbf{x}, t)$, which is obtained from the Fokker-Planck equation equivalent to eqn (12) (see, e.g. Gardiner, 1983, section 5):

$$\dot{P}(\mathbf{x}, t) = -\frac{\partial}{\partial \mathbf{x}} \mathbf{d}(\mathbf{x}, t) P(\mathbf{x}, t) + \frac{1}{2} \frac{\partial^2}{\partial \mathbf{x} : \partial \mathbf{x}} . \mathbf{Q} P(\mathbf{x}, t) \tag{15}$$

where $\mathbf{d}(\mathbf{x}, t) = \mathbf{f}_{\text{intr}}(\mathbf{x}) + \theta(t - t_0) c_{\text{intent}} \mathbf{f}_{\text{intent}}(\mathbf{x})$ is the drift vector and $\mathbf{Q} = \mathbf{q} \, . \, \mathbf{q}^T$ (T denotes the transposition) is the diffusion matrix. This Fokker-Planck equation can be solved—if only numerically—using an initial distribution that is determined as a stationary solution of a local model of the state $\mathbf{x}_1$ (cf. above). In terms of the solution $P(\mathbf{x}, t)$ we may then define the *probability weight of the intended state* $\mathbf{x}_2$ *as*

$$P_U(t) = \int_{\mathbf{x} \in U} P(\mathbf{x}, t) \, \mathrm{d}\mathbf{x} \tag{16}$$

where U is a neighborhood of $\mathbf{x}_2$ and $\mathrm{d}\mathbf{x}$ denotes a volume element in the space of collective variables $\mathbf{x}$. The rate of change $v(t)$ of this probability weight

$$v(t) = \mathrm{d}P_U(t)/\mathrm{d}t \tag{17}$$

can be interpreted as the velocity of the switching process. In particular, when $v(t)$ is positive, it is the probability that the process enters U in the interval $[t, t + \mathrm{d}t]$. Thus, if we define *switching time* as the time at which the process first enters the neighborhood U of the intended state, then this time is a random variable whose probability density is given by $v(t)$ and whose mean is:

$$\tau_{\text{switch}} = \int_0^\infty t \, \mathrm{d}P_U(t). \tag{18}$$

We refer to the quantity τ_{switch} as the *mean switching time*. Whether $v(t)$ is positive at all times depends on the initial conditions chosen and the strength of the attraction to the intended state.

In physical systems a similarly defined switching time has been used to characterize transients in non-equilibrium phase transitions (e.g. Broggi & Lugiato, 1984, for transients in a laser system). We have used this measure before (Schöner *et al.*, 1986) to characterize transients that occur when one mode of movement co-ordination changes spontaneously to another, a phenomenon identified in detail as a non-equilibrium phase transition (see section 2).

In the present context, switching time may be the most appropriate measure of the intentional switching process in which both the initial and the final (intended) state are specified. The mathematical definition of switching time suggests an experimental measure to estimate τ_{switch}: assume a system, initially prepared in one behavioral pattern, intentionally changes its behavioral pattern. Measuring the behavioral pattern in terms of collective variables we can define criteria in the collective variable space for when the new pattern is reached and for when the transient begins. These criteria correspond to defining the neighborhood U above. The time between these events is a sample estimate of switching time. Several runs of such intentional switching may be viewed as an ensemble that determines the distribution of the switching time estimate and its mean.

In so far as the probability $P(\mathbf{x}, t)$ depends on the intrinsic dynamics $\mathbf{f}_{intr}$, it also depends on the nature of the stable states $\mathbf{x}_1$ and $\mathbf{x}_2$. In general, we expect states with different stability or with basins of attraction differing in size to exhibit different evolutions of this probability. In particular, consider the case where $\mathbf{x}_1$ and $\mathbf{x}_2$ differ in their stability, for example, where $\mathbf{x}_1$ is more stable than $\mathbf{x}_2$. The process of switching from $\mathbf{x}_1$ to $\mathbf{x}_2$ will be different in terms of switching time from the process of switching in the opposite direction, i.e. the former will be slower than the latter. In this way the intrinsic dynamics may be felt during an intentional change of behavior. Below, this aspect is demonstrated explicitly in a concrete example.

5.2. FIRST EXIT TIMES

To define a first exit time we consider a neighborhood V of the initial state $\mathbf{x}_1$. The first exit time is defined as the time when the system first leaves this neighborhood. This time is a random variable that depends on the point in V at which the process starts. The distribution of the first exit time can, in principle, be determined from the Fokker-Planck eqn (15) (see, e.g. Gardiner, 1983, section 5.4). More specifically, this distribution can be obtained by solving a partial differential equation in which the starting point provides the spatial dimension. The *mean first exit time* can then be shown to obey a time-independent partial differential equation of the form:

$$\mathbf{d}(\mathbf{x}) \cdot \frac{\partial}{\partial \mathbf{x}} T(\mathbf{x}) + 2\mathbf{Q} \cdot \frac{\partial^2}{\partial \mathbf{x} : \partial \mathbf{x}} T(\mathbf{x}) = -1 \qquad (19)$$

for $\mathbf{x} \in V$. For our system (eqn (12)) this equation is valid only for $t \geq t_0$, where the process is homogeneous. On the boundary of V we require $T(\mathbf{x}) = 0$. This equation is somewhat difficult to solve in more than one dimension. In the next section we shall use the exact solution for one dimension.

The concept of first exit time, or, in one dimension, first passage time, has been used for a long time in systems with multiple stable states. In chemical reaction kinetics, Kramers (1940) first introduced this idea to derive the Arrhenius law. In the present context the definition of first exit time suggests an experimental measure similar to switching time. Assume a behavioral pattern has been established at time $t=0$. Assume also that without change of conditions a spontaneous change of pattern occurs. In terms of the collective variables characterizing the pattern we determine a criterion as to when the pattern has been abandoned, which corresponds to choosing the region V. The time at which this criterion is first fullfilled is a sample estimate of the first exit time. This measure is useful when the pattern that is assumed is not unique. An example may be the spontaneous changes of pattern that can be observed over a very long time. Various first exit times can then provide estimates of the relative stability of the different patterns available (a concrete example would be the analysis of the relative stability of the different gaits in freely roaming horses, where this method of analysis may supplement the data on the distribution over the different gaits over a long time (Hoyt & Taylor, 1981)). For intentional switching mean first exit time may not be entirely adequate, because (a) the final state is determined and (b) there is a change of parameter at the onset of intention that is difficult to determine other than through the exit criterion itself. However, the present discussion enables us to introduce a third concept called *first entry time.*

5.3. FIRST ENTRY TIMES

First entry times can be defined in a fashion completely analogous to first exit time. In fact, by looking at the complement U of a neighborhood V of the final state, we may carry over the definition of first exit time to this case. This trick also allows for the calculation of these times from the Fokker–Planck equation. However, it is necessary to assume an initial distribution in U.

The experimental meaning of first entry times is obviously the following. We need to identify when the switching begins in a system that intentionally switches from an unspecified initial state to a certain behavioral pattern. A criterion in the collective variable space can be used to determine when the new state is established. The time between when switching begins and when the new state is reached is an estimate of the first entry time. This measure can be used when an intentional change of behavior to the same final state occurs from a number of different initial states.

Note that for the case of a single order parameter, x_i, the concepts of first entry and exit times are identical due to the topology of the line. In that frequently studied case the phrase *first passage time* is used. Generalizing these measures to other attractor types, for example, to limit cycles, is actually not straightforward mathematically. We restrict ourselves to a qualitative discussion in the context of a concrete oscillator system (section 7).

6. Intentional Switching of Modes of Movement Co-ordination: Relative Phase Dynamics

Recently, the model system summarized briefly in section 2 has been used by Kelso & Scholz to study experimentally intentional changes of behavioral pattern (see Schöner & Kelso, 1988*d*; Kelso *et al.*, in press, for details). Again, the task was rhythmic bi-manual movement starting either in an in-phase or an anti-phase co-ordination pattern. For the first ten cycles the movement was paced by a metronome, which was then turned off. The subject's instruction was to continue the oscillation at the initial frequency. After a further ten cycles a signal indicated to the subject that she or he should switch as quickly as possible to the other mode of co-ordination. Using interactive computer displays of the trajectories and the calculated relative phase between them, the length of the actual transient between the two modes of co-ordination was determined. The mean of these durations is shown in Fig. 1 as a function of the various oscillation frequencies used. Clearly, switching from the in-phase to the anti-phase mode is slower by approximately a factor of two than switching in the opposite direction. Figure 2 shows the experimental distributions of the switching times in the two directions. Note the longer tail and larger mean of the switching time in the in- to anti-phase condition.

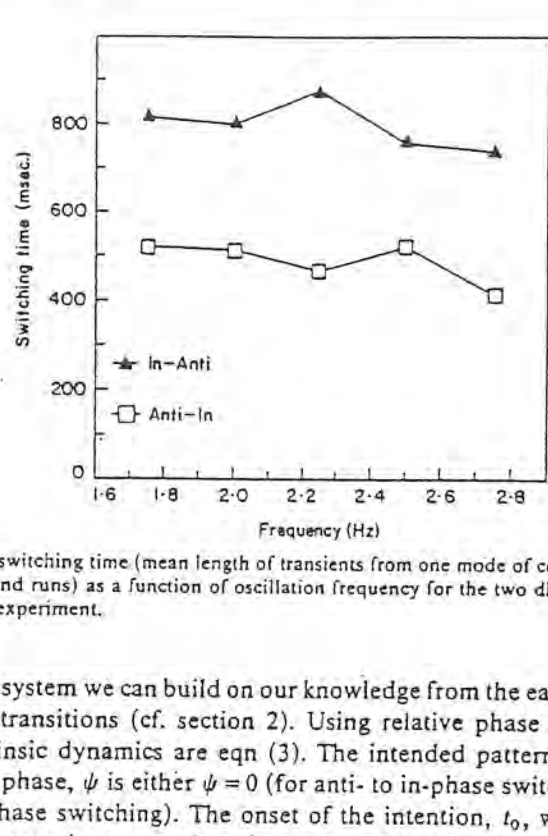

FIG. 1. The mean switching time (mean length of transients from one mode of co-ordination to other, mean over subjects and runs) as a function of oscillation frequency for the two directions of switching in Kelso & Scholz's experiment.

Modelling this system we can build on our knowledge from the earlier experiments on spontaneous transitions (cf. section 2). Using relative phase as the order parameter, the intrinsic dynamics are eqn (3). The intended pattern, defined by the intended relative phase, ψ is either $\psi = 0$ (for anti- to in-phase switching) or $\psi = \pm\pi$ (for in- to anti-phase switching). The onset of the intention, t_0, will be viewed as the time when the transient starts. In a simple approximation we assume a perturbation of the form:

$$-\theta(t - t_0)c_{\text{intent}} \sin(\phi - \psi) \tag{20}$$

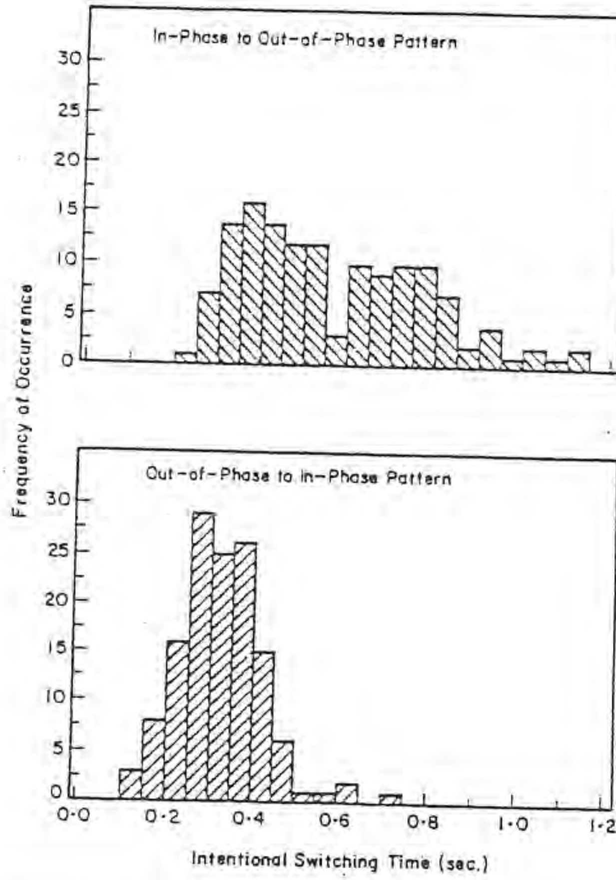

FIG. 2. The frequency distributions of switching times (all subjects and runs) for the two directions of switching in Kelso & Scholz's experiment.

that attracts relative phase, ϕ to intended relative phase, ψ with strength $c_{\text{intent}} > 0$. Here, as discussed above, we have assumed an abrupt onset of the intention in the form of a Heaviside step function $\theta(t-t_0)$ (cf. eqn (14)) as a first approximation. The complete model is thus:

$$\dot{\phi} = -a \sin(\phi) - 2b \sin(2\phi) - \theta(t-t_0) c_{\text{intent}} \sin(\phi - \psi) + \sqrt{Q}\xi_t. \tag{21}$$

To characterize the transient switching process we calculate the switching time distribution (Appendix A) as well as mean first passage times (Appendix B). Figure 3 shows the mean switching times for the two directions of switching calculated along a path in the parameter plane of the intrinsic dynamics, that was shown in earlier work (Schöner *et al.*, 1986) to correspond to an increase in oscillation frequency. Importantly, the starting point to the left corresponds to parameter values for which both states at $\phi = 0$ and $\phi = \pm\pi$ are equivalent in the intrinsic dynamics, while the state $\phi = \pm\pi$ gradually becomes less stable along the parameter path to

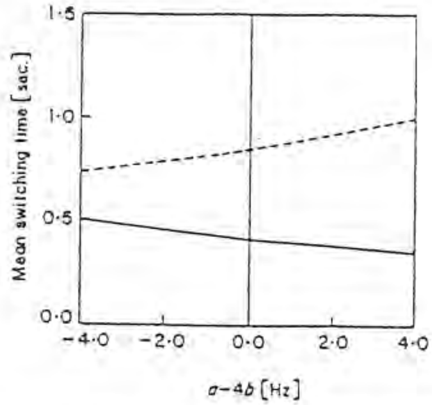

FIG. 3. The mean switching times (eqn (A6)) as calculated from the theoretical model eqn (21) along the path $a+4b=4$ Hz in the parameter plane, (a, b) of the intrinsic dynamics. From left to right the intrinsic stability of the anti-phase pattern decreases, such that it becomes intrinsically unstable at $a-4b=0$ Hz. Note that the mean switching time for switching from in-phase to anti-phase (dashed line) is much larger than for switching from anti-phase to in-phase (solid line) (for details see Appendix A).

the right. At $a-4b=0$ the anti-phase state looses stability in the intrinsic dynamics. Note the larger mean switching time for switching from the more stable to the less stable state, a difference that becomes more pronounced, the more the two states differ in stability. This result demonstrates that the difference in mean switching time is due to the different relative stability of the two states as discussed for the general case in section 5. Figure 4 shows the probability distributions of switching time for the two directions of switching. These distributions were calculated at parameter values of the intrinsic dynamics that correspond to a realistic situation in the bistable régime (cf. Schöner *et al.*, 1986). Comparison with the experimental

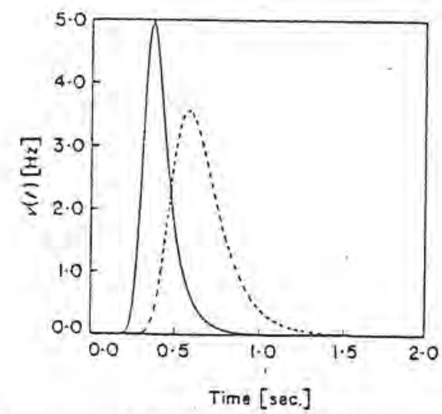

FIG. 4. The theoretical distributions of switching time (eqn (A5)) calculated at the parameter values $a=2$ Hz and $b=0{\cdot}5$ Hz of the intrinsic dynamics for switching from anti-phase to in-phase (solid) and from in-phase to anti-phase (dashed). Note the sharper peak and shorter tail for switching to the more stable pattern (in-phase).

figure reveals that the agreement is quite striking, both qualitatively and (within the inherent bounds of accuracy) quantitatively. We want to emphasize that only one new parameter, c_{intent} is adjusted, while all parameters of the intrinsic dynamics are supplied from studies of the underlying co-ordination system.

The mean first passage times for passage from $\phi = 0$ to $\phi = \pm\pi$ in one case ($\psi = \pi$) and for the opposite direction in the other case ($\psi = 0$) are shown in Fig. 5 evaluated along the same path in the parameter plane of the intrinsic dynamics (see Appendix B for the calculations). The mean first passage time exhibits the same dependence on direction of switching as the switching time, i.e. first passage from the less stable state to the more stable state occurs, on average, earlier than vice versa.

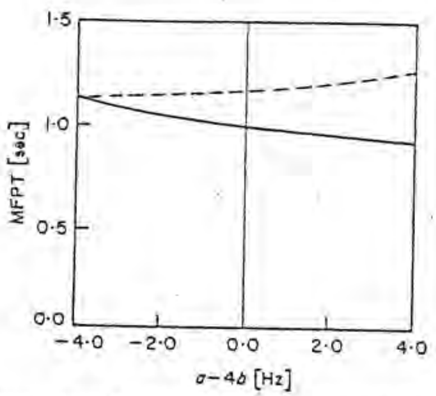

FIG. 5. The mean first passage times for passage from anti-phase to in-phase (eqn (B1): solid line) and for passage from in-phase to anti-phase (eqn (B2): dashed line) along the same path $a+4b=4$ Hz used in Fig. 3. At the left ($a = 0$ Hz) both states are equivalent and the mean first passage times are independent of direction of switching. To the right the anti-phase state becomes intrinsically less stable and the mean first passage time shows a direction dependence similar to the mean switching time, with passage from in-phase to anti-phase taking longer on average than passage from anti-phase to in-phase (for details see Appendix B).

2

Note that the intentional dynamics eqn (20) can change the stability of a pattern from being intrinsically unstable to stable. This is obvious in the right part of Fig. 3 (for $a-4b>0$) where the anti-phase pattern is intrinsically unstable, but is stabilized by the intentional contribution. This is an example of qualitative change of dynamics due to intention.

7. Intentional Switching of Modes of Movement Co-ordination: Oscillator Dynamics

Due to our knowledge about the individual oscillators (see section 2) we may easily address intentional switching among modes of co-ordination also on the component level. With endeffector positions x_i and velocities $\dot{x}_i$ of each limb ($i = 1, 2$) as collective variables, we adopt eqn (5) to describe the intrinsic dynamics on this level of description. The intentional dynamics are determined as before by the intended relative phase, ψ, the (abrupt) onset of intention at time t_0, and the strength of the intention relative to the intrinsic dynamics. As we study switching from one intrinsic pattern to another, the functional form of the intrinsic coupling should be sufficient to define also the attraction to the intended pattern. In fact, due to

$$\sin(\phi - \pi) = -\sin(\phi) \tag{22}$$

the linear coupling term will be sufficient as an intentional perturbation of the oscillator dynamics:

$$g_{\text{intent}}(\dot{x}_1, \dot{x}_2) = -\theta(t - t_0)c_{\text{intent}} \cos(\psi)(\dot{x}_1 - \dot{x}_2) \tag{23}$$

where $c_{\text{intent}} > 0$ is as before and θ is again the step function. For $\psi = 0$ and thus $\cos\psi = 1$ this coupling stabilizes the in-phase pattern, while for $\psi = \pi$ and thus $\cos\psi = -1$ it stabilizes the anti-phase pattern. The complete dynamics are thus:

$$\begin{aligned} \ddot{x}_1 + f(x_1, \dot{x}_1) &= g_{\text{intr}}(x_1, \dot{x}_1; x_2, \dot{x}_2) + g_{\text{intent}}(\dot{x}_1, \dot{x}_2) \\ \ddot{x}_2 + f(x_2, \dot{x}_2) &= g_{\text{intr}}(x_2, \dot{x}_2; x_1, \dot{x}_1) + g_{\text{intent}}(\dot{x}_2, \dot{x}_1) \end{aligned} \tag{24}$$

where g_{intr} is defined as in eqn (5).

In the limit of weak non-linearity $|\alpha| \ll \omega$ we can employ standard techniques, that is, the rotating wave approximation and the slowly varying amplitude approximation (see, for example, Haken, 1985; Haken *et al.*, 1985) to eliminate the amplitudes in these equations. In fact, as the functional form of the intentional coupling is identical to that of the intrinsic coupling, we may absorb the new terms into redefinitions of the parameters of the original Haken *et al.* (1985) model. The result of the calculation is:

$$\dot{\phi} = -a\sin(\phi) - 2b\sin(2\phi) - \theta(t - t_0)c_{\text{intent}}\cos(\psi)\sin(\phi) \tag{25}$$

where $a = -k - 2lr_0^2$ and $b = lr_0^2/2$ with $r_0^2 = -\alpha/(3\beta\omega^2 + \gamma)$. Here polar co-ordinates (r_i, ϕ_i) have been introduced for the two oscillators through $x_i(t) = 2r(t)\cos(\omega t + \phi_i(t))$, $(i = 1, 2)$ and the relative phase is defined by $\phi = \phi_2 - \phi_1$. Using a trigonometric identity we find

$$\dot{\phi} = -a\sin(\phi) - 2b\sin(2\phi) - \theta(t - t_0)c_{\text{intent}}\sin(\phi - \psi) \tag{26}$$

which is exactly the deterministic part of our model eqn (21) on the relative phase level.

A word of caution must be added to the foregoing derivation. When we are interested in the transient switching process it is not clear *a priori* to what extent the slowly varying amplitude approximation holds up. We therefore also analyze the oscillator dynamics directly. Indeed, a new aspect needs to be addressed to perform this step properly, namely the fact that the coupling functions g_{intr} and g_{intent} always alter the properties of the individual oscillators (mathematically it may therefore be more appropriate to discuss the oscillator dynamics in the four dimensional phase space $(x_1, \dot{x}_1, x_2, \dot{x}_2)$. Such analysis of coupled oscillator systems is surprisingly recent (see, e.g. Aronson *et al.*, 1987)). We restrict ourselves here to a qualitative discussion.

We may introduce an effective linear damping constant

$$\alpha_{\text{eff}} = \alpha - k + \theta(t - t_0)c_{\text{intent}} \cos(\psi) < 0 \tag{27}$$

that roughly accounts for the effects of the linear coupling terms on the oscillator amplitude. For the weak non-linearity limit we need

$$|\alpha_{\text{eff}}| \ll \omega. \tag{28}$$

Our approach is thus to keep α_{eff} constant when the intentional coupling is turned on. Thus the linear damping term of the oscillator function f should be modified to

$$\alpha_{\text{intent}} = \alpha - c_{\text{intent}} \cos(\psi)\theta(t - t_0). \tag{29}$$

Using this modification we have solved the oscillator equations numerically (Appendix C) in a number of cases. Figure 6 shows a typical run that starts out close to the anti-phase condition, to which the oscillators relax quickly. At $t = 10$ sec. an intentional coupling of relative strength $-c_{\text{intent}}/k = 10$ is turned on with $\psi = 0$. Within approximately 3 sec. the oscillators switch to in-phase. Figure 7 shows the opposite situation at the same parameter values, in which the switching takes approximately 5 sec. and thus is slower. These simulations show, more qualitatively,

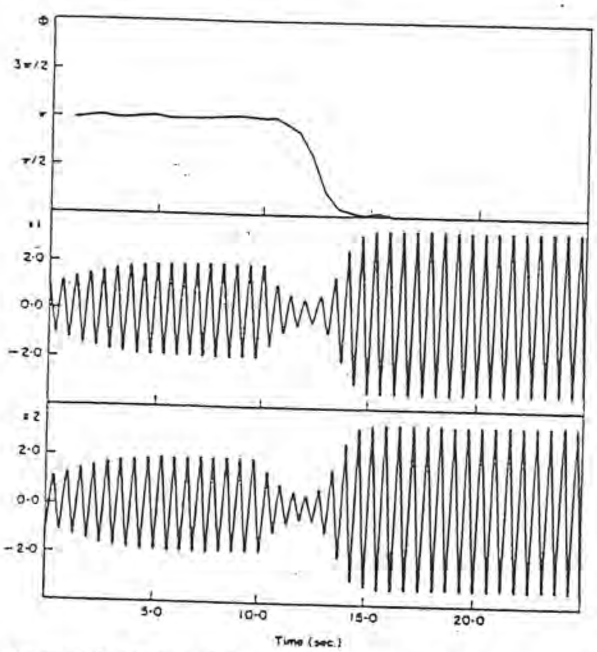

FIG. 6. Results of a numeric solution of the oscillator model eqn (24). The middle trace shows the position, x_1 of the first oscillator, the lower trace the position, x_2 of the second oscillator. In the top trace the relative phase of the two oscillators (calculated as peak to peak latency divided by cycle time) is depicted. At the chosen parameter values both in-phase and anti-phase locking are intrinsically stable (see Appendix C for details). The oscillators are initially close to an anti-phase locking, to which they relax quickly. At $t = t_0 = 10$ sec. The intentional perturbation eqn (23) becomes active with $\psi = 0$ and the oscillators switch to in-phase within approximately 3 sec.

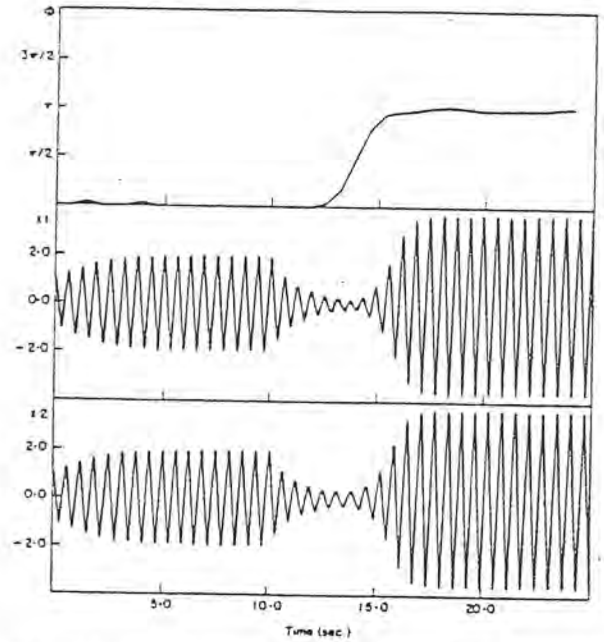

FIG. 7. The oscillator model eqn (24) solved at the same set of parameter values as Fig. 6. Initially the oscillators are close to an in-phase locking, to which they relax quickly. At $t = t_0 = 10$ sec. the intentional perturbation becomes active with $\psi = \pi$. The oscillators then switch to anti-phase locking within approximately 5 sec.

the influence of the relative stability of the intrinsic patterns on the intentional switching process at the oscillator level of description. Furthermore, it is obvious that the intentional perturbation, eqn (23), can stabilize one pattern and destabilize another, an instance of the qualitative change of dynamics due to intentional information.

8. Summary and Discussion

In this article we extended the dynamic pattern theory of behavioral patterns to include intentional changes of behavior. The key idea was to treat the goal of an intention to change behavior in terms of behavioral information, which was defined as a part of the pattern dynamics that attracts the behavioral variables toward the intended behavioral state. This approach provided basically two general predictions. First, the intrinsic dynamics—the dynamics of the behavioral patterns in the absence of any intention to change behavior—should affect intentional changes of behavioral pattern. Second, the contribution of intention to the pattern dynamics can alter the dynamics of behavioral patterns, both quantitatively and qualitatively. The latter amounts to changing the stability of such patterns. Switching time, first exit and first entry times were discussed as measures of the transient switching process in behavioral change. The influence of the intrinsic dynamics on intentional switching is predicted to be manifest in the switching time measure.

The theory was applied to a well-studied experimental model system involving human bi-manual movement co-ordination. Here intentional change of mode of co-ordination was modelled explicitly on the level of relative phase as a collective variable. It was possible to spell out the general predictions in detail, including explicit calculation of the switching time distribution and the mean first passage time. In this system the differential stability of the two modes of co-ordination leads to mean switching times that depend strongly on the direction of switching between the two modes. Recent experiments (Kelso *et al.*, in press) demonstrated the influence of intrinsic dynamics on intentional switching. Moreover, considerable agreement between the explicit model and the experimental results was found.

The theory was also implemented on the level of the component oscillators from which it was possible to derive the dynamics of the relative phase level, thus linking again different levels of observation. Direct solution of the oscillator equations also exhibited the switching phenomenon, in addition to (more qualitatively) the direction dependence of switching time.

Switching time and measures of first exit or entry times may be of quite general relevance for behavioral analysis. Thus it may be possible, for example, to use these measures of behavioral change in cases where the intrinsic dynamics are not known. The measures then provide information on the relative stability of the underlying patterns. In this way intentional behavioral changes may be used quite generally to probe the underlying behavioral dynamics. The second general result is conceptually important: the feature that intention may change the dynamics of behavioral patterns, in particular, stabilize or destabilize them, resolves a sometimes discussed dichotomy between intentionality and dynamic laws of behavior (see e.g. comments on Kelso *et al.*, 1986). Thus a behavioral task or function, which defines a certain dynamic pattern law, may be viewed as the result of an intentional change of behavior and thus be given meaning within a dynamic pattern description. It is important to stress, however, that this view has predictive power only when the dynamics of the underlying behaviors, i.e. the intrinsic dynamics, are known.

In how far does the present work contribute towards answering the questions posed at the outset? We would like to point out three aspects: first, by defining the goal of an intentional change of behavior in terms of the collective variables that characterize the behavioral patterns we provide an operational language with which to specify what can be intended. Only when the collective variables capture all aspects of the behavior that are relevant to the intentional behavioral change can this language be successfully used. While this restricts the range of the theory it also suggests the inclusion of other aspects relevant to an intention in the behavioral pattern description by enlarging the collective variable space. Second, we expressed the effect of an intention to change behavior on the level of the behavioral pattern dynamics. Thus measures of these dynamics such as switching time, relaxation time or fluctuations provide methods to experimentally assess the influence of an intention on the behavioral patterns. These methods can be used to address the question of how intentions are constrained by the behavioral pattern dynamics. Third, by making and testing predictions we learned about the nature of the process of intentional behavioral change.

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APPENDIX A

The starting point is the Fokker–Planck equation corresponding to (21) which reads:

$$\dot{P}(\phi, t) = \frac{\partial}{\partial \phi}\left[a \sin(\phi) + b \sin(2\phi) + \theta(t - t_0) c_{\text{intent}} \sin(\phi - \psi)\right] P(\phi, t) + \frac{1}{2} Q \frac{\partial^2}{\partial \phi^2} P(\phi, t). \tag{A1}$$

This equation is solved numerically using a forward-time-centered-space integration algorithm that is stable for adequate ratio of time and space step (see, e.g. Press *et al.*, 1986). In integrating the distribution has to be normalized at each time step since the norm of the distribution is marginally stable in the Fokker–Planck dynamics. As initial conditions we choose stationary solutions of local models of the in-phase or anti-phase modes corresponding to the direction of switching. These distributions are (see Schöner *et al.*, 1986):

$$P_{\text{init}}(\phi) = \frac{g}{\sqrt{\pi}\, \operatorname{erf}(\pi g)} \exp\left[-g^2 \phi^2\right] \tag{A2}$$

with $g = \sqrt{(4b + a)/Q}$ for in-phase and

$$P_{\text{init}}(\phi) = \frac{h}{\sqrt{\pi}\, \operatorname{erf}(\pi h)} \exp\left[-h^2 (\pi - |\phi|)^2\right] \tag{A3}$$

with $h = \sqrt{(4b - a)/Q}$ for anti-phase (erf denotes the error function). Here the parameters a_{init} and b_{init} were always chosen in the same way $a_{\text{init}} = b_{\text{init}} = 1$ Hz corresponding to a state in the bi-stable régime of the intrinsic dynamics (cf. Schöner *et al.*, 1986). This accounts for the small difference in the mean switching times at $b = 0$ (left end of abscissa in Fig. 3) where both states at $\phi = 0$ and $\phi = \pm\pi$ are dynamically equivalent.

The probability weight of the new state at $\phi = \psi$ can be defined as

$$P_\delta(t) = \int_{\psi - \delta}^{\psi + \delta} \mathrm{d}\phi\, P(\phi, t) \tag{A4}$$

for adequately chosen $\delta > 0$. The switching time distribution is then

$$v(t) = \frac{\mathrm{d}P_\delta(t)}{\mathrm{d}t} = \int_{\psi - \delta}^{\psi + \delta} \mathrm{d}\phi\, \dot{P}(\phi, t) \tag{A5}$$

and the mean switching time is

$$\tau_{\text{switch}} = \int_0^\infty t\, \mathrm{d}P_\delta(t). \tag{A6}$$

In the numerical work we used $\delta = \pi/10$ which corresponds to a criterion interval for relative phase of roughly two typical experimental standard deviations (cf. Kelso *et al.*, 1986).

Both the switching time distribution $v(t)$ and the mean switching time τ_{switch} are determined from the result of a numerical integration of eqn (A1). The parameters for the intrinsic dynamics were varied along the path $4b + a = 4$ Hz in the (a, b)-plane that was shown in earlier work to correspond roughly to a scaling of frequency from low to high frequencies (see Schöner *et al.*, 1986, Fig. 4). The noise level was chosen as in the earlier work as $Q = 0{\cdot}25$ Hz. The only adjustable parameter c_{intent} was then chosen as $c_{intent} = 10$ Hz to account roughly for the observed order of magnitudes of the mean switching times. This choice implies an approximate relative strength of intentional to intrinsic dynamics of $c_{intent}/(4b + a) = 2{\cdot}5$. Interestingly this relative strength of behavioral information dynamics to intrinsic dynamics is very similar to the values found to realistically cover the influence of environmental information (Schöner & Kelso, 1988*a*, where $c_{info}/(4b + a)$ was between 2 and 4).

APPENDIX B

In one dimension the differential equation eqn (19) can be exactly solved. We use a special trick, namely erecting a reflecting boundary at $\phi = \pi$ for switching to 0 and at $\phi = 0$ for switching to π. This way a problem of large numerator and denominator can be avoided. The resulting expressions for the mean first passage times are (see Gardiner, 1983, section 5.2):

$$T(\pi \to 0) = \frac{2}{Q} \int_0^{\pi} \frac{d\phi}{A(\phi)} \int_{\phi}^{\pi} A(\phi')\, d\phi' \tag{B1}$$

and

$$T(0 \to \pi) = \frac{2}{Q} \int_0^{\pi} \frac{d\phi}{A(\phi)} \int_0^{\phi} A(\phi')\, d\phi' \tag{B2}$$

where

$$A(\phi) = \exp\left[-2(a + b + c_{intent} \cos \psi)/Q\right] \times \exp\{2[a \cos(\phi) + b \cos(2\phi) + c_{intent} \cos(\phi - \psi)]/Q\}. \tag{B3}$$

These integrals were determined by numerical double integration using two Romberg procedures. The choice of model parameters was as for switching time (Appendix A). The mean first passage time, as calculated here, assumes a point-like initial distribution with the system starting at $\phi = 0$ and $\phi = \pm\pi$ respectively, almost surely. Thus no parameters for the initial distributions are left free. In particular, as these point distributions are equivalent for $\phi = 0$ and $\phi = \pm\pi$, the mean first passage times for the two directions of passage are identical when these two states are equivalent dynamically (left side of abscissa in Fig. 5).

APPENDIX C

A Runge-Kutta procedure was used to solve the oscillator eqns (23) numerically. The time step was chosen as $\Delta t = 0{\cdot}01$ sec. Two independent gaussian white noise processes of variance $Q = 0{\cdot}1$ Hz were added, one to each oscillator equation, to probe the stability of calculated solutions.

The model parameters were chosen following the same strategy as for the relative phase dynamics (section 6), that is, all parameters of the intrinsic dynamics (eqn (5)) were chosen in accordance with earlier work on rhythmic single limb movement (Kay *et al.*, 1987) and on the spontaneous transition in co-ordinated movement (Haken *et al.*, 1985). Specifically we chose: $\alpha = -1$ Hz, $\omega = 10$ Hz, $\beta = 0{\cdot}0001$ sec., $\gamma = 1$ Hz, $k = -0{\cdot}1$ Hz, $l = 0{\cdot}025$ Hz. Then $|\alpha_{\text{eff}}| = 0{\cdot}9$ Hz $\ll \omega$. These values correspond to a co-ordination pattern in the bi-stable régime. The only adjusted parameter was the strength of the intentional coupling c_{intent}. This was chosen as $c_{\text{intent}} = 2$ Hz to provide for reasonably fast switching.